MECA-H303 Vibration TP1

Kinematics & Statics

Simple lift mechanism

For the following mechanism, we ask you to:

1. Calculate the number of degrees of freedom
2. Write the constraint equations
3. Using the principle of virtual work, express the ratio between F_x and F_y as a function of the system's coordinates, when the system is in static equilibrium.

Additional details for the exercise:

- Use the coordinate system centered on point C
- Lengths $|AE| = |BE| = |CE| = |DE| = L$
- The mass of all bars is negligible
- The final answer should be expressed as a function of the coordinates on the schematic

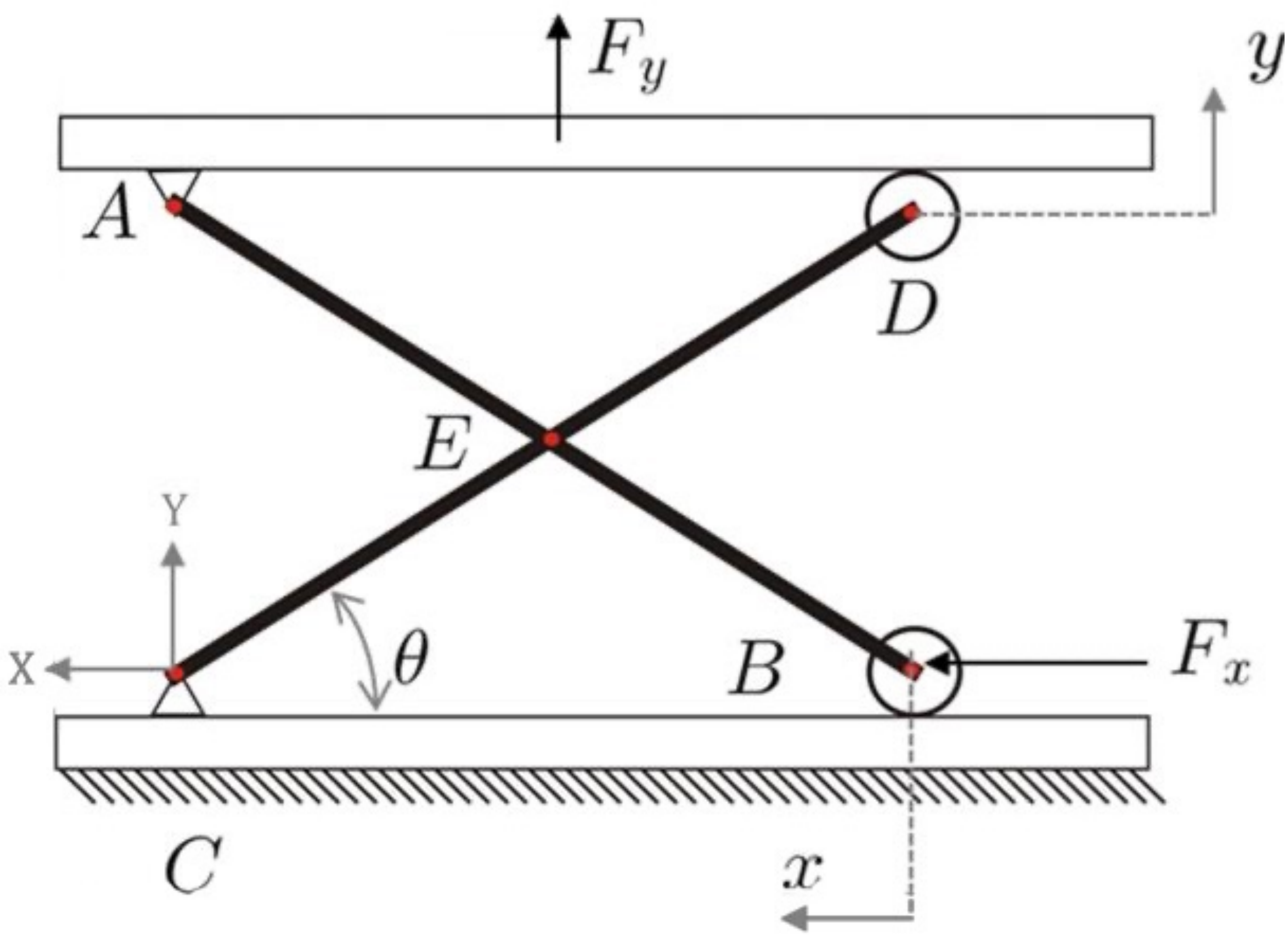


Figure 1: simple lift mechanism

1

n : n° of bodies

l : n° of joints

v_i : n° of dofs at joint i

$b = l - n = 5 - 3 = 2$ (n° of closed loops)

$$N = \left(\sum_{i=1}^{l} v_i\right) - 3b$$

$$N = 1 + 2 + 1 + 2 + 1 - 3.2 = 1 \quad \text{(n° of dofs)}$$

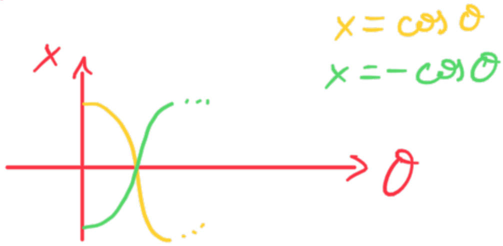
joint: A B C D E

2

$$x_B = -2L\cos\theta$$

$$y_D = 2L\sin\theta$$

when $x \nearrow$ then $\theta \nearrow$



3

$$d\overline{\tau} = 0$$

$$\Leftrightarrow d\overline{F_x} + d\overline{F_y} = 0$$

$$\Leftrightarrow F_x\, dx_B + F_y\, dy_A = 0$$

$$\begin{cases} dx_B = 2L\, d\theta \sin\theta \\ dy_D = 2L\, d\theta \cos\theta \end{cases}$$

$$\Leftrightarrow \frac{F_y}{F_x} = \frac{-dx_B}{dy_D} = \frac{-\sin\theta}{\cos\theta} = -tg\,\theta$$

F_x and F_y need to be of opposite signs to have the lifting plate in static equilibrium

Simplified crane

For the following mechanism, we ask you to:

1. Calculate the number of degrees of freedom
2. Write the constraint equations
3. Using the principle of virtual work, express the ratio between the mass m and the applied torque M for the system to be in static equilibrium

Informations additionnelles :

- The long bar has a length of $2l$
- The short bar has a length of l and is connect with a hinge to the middle of the long bar
- The mass m is fixed to the end of the long bar
- The mass of all bars is negligible
- The final answer should be expressed as a function of the coordinates on the schematic

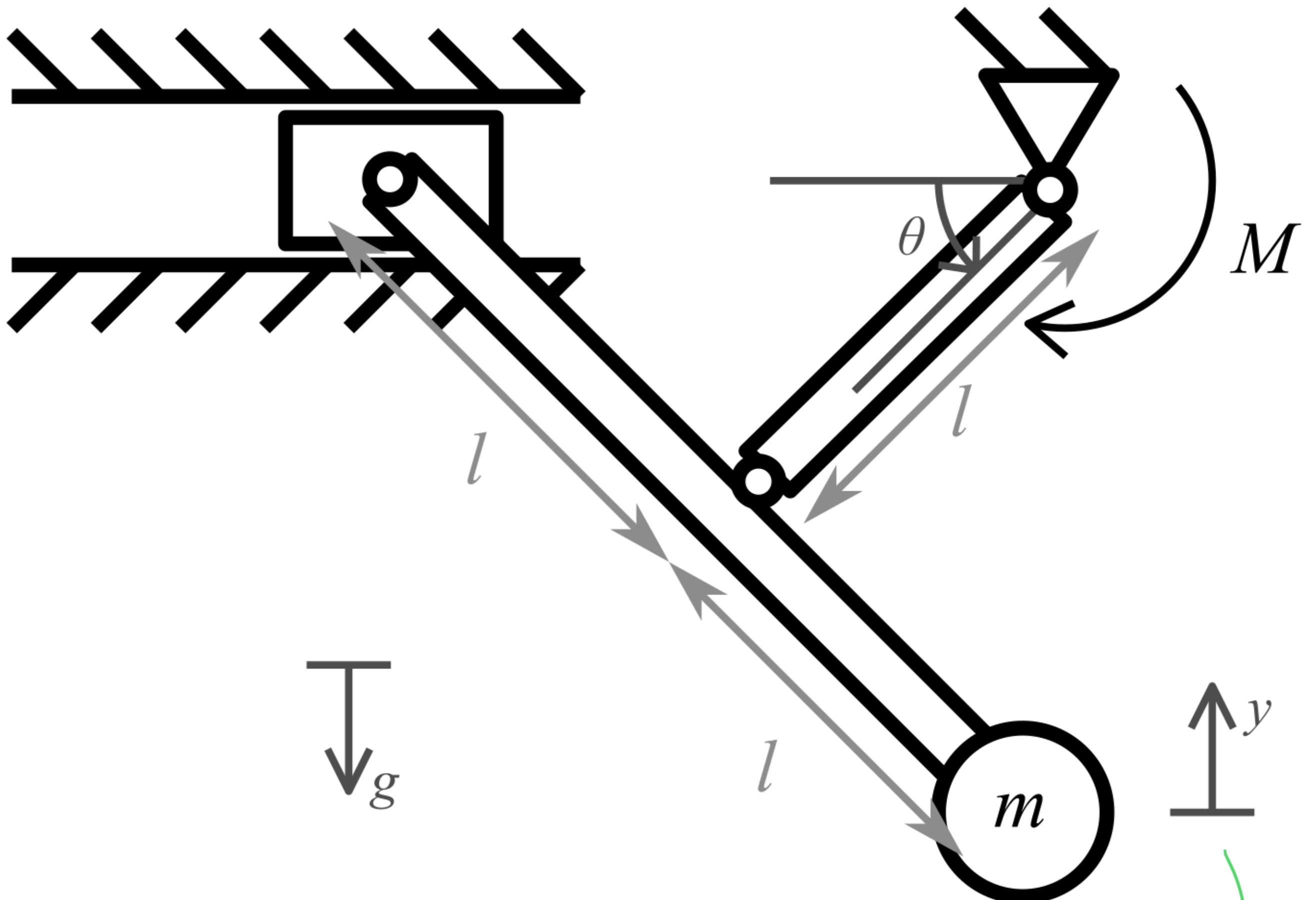


Figure 2: simplified crane

global coordinates not attached to the mass, but representing the height of the mass

$\boxed{1}$

$$b = l - n = 4 - 3$$

l: n° joints, n: n° bodies

$$N = \left(\sum_{i=1}^{l} \nu_i\right) - 3b$$

$$N = \underbrace{2}_{A} + \underbrace{1}_{B} + \underbrace{1}_{C} + \underbrace{0}_{D} - 3 \cdot (4-3) = 1$$

Also acceptable : $N = 2 + 1 + 1 - 3(3-2) = 1$ (mass rigid with AD)

Also acceptable : $N = 1 + 1 + 1 + 1 + 0 - 3(5-4) = 1$ (sep. slider)

Also acceptable : $N = 1 + 1 + 1 + 1 - 3(4-3) = 1$ (mass rigid and sep. slider)

$\boxed{2}$

$$y = -2l \sin\theta$$

$$d_y = -2l\cos\theta \, d\theta$$

$\boxed{3}$

$m_D g$ is opposite to y — M is opposite to θ

$$d\bar{\tau} = 0 \iff d\bar{F}_y + d\bar{M}_\theta = 0 \iff -F\, d_y - M\, d\theta = 0$$

$$m g d_y = -M \, d\theta \iff \frac{m}{M} = \frac{-d\theta}{d_y}\frac{1}{g}$$

$$\frac{m}{M} = \frac{1}{2l\cos\theta}\,\frac{1}{g}$$

(verification through moment equilibrium $\sum M_A = 0 \iff M_B - m_D g 2l\cos\theta = 0$)

Crank and slider

For the following mechanism, we ask you to:

1. Calculate the number of degrees of freedom
2. Write the constraint equations
3. Using the principle of virtual work, express the ratio between the force F and the torque C applied as a function of the system's coordinates, when the system is in static equilibrium.

Additional details for the exercise:

- DC is one rigid bar pinned in point P
- The final answer should be expressed as a function of the coordinates on the schematic

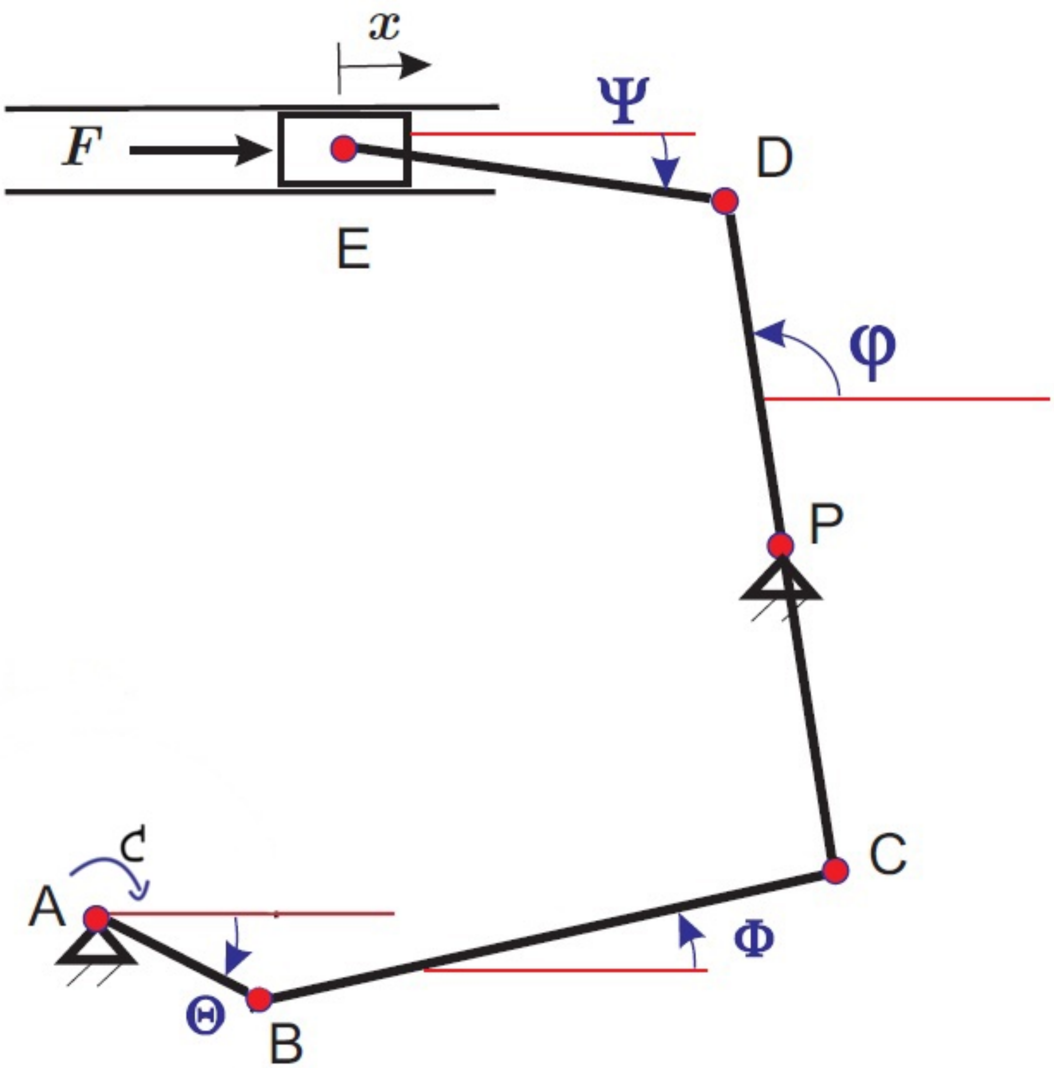


Figure 3: Crank and slider

$\boxed{1}$

$$N = \sum_{i=1}^{l} \nu_i - 3b$$

ν_i : n° of dofs at joint i

b : n° of closed loops

$= l - n = 6 - 3 = 2$

n : n° of bodies

l : n° of joints

$$N = \underset{A}{1} + \underset{B}{1} + \underset{C}{1} + \underset{P}{1} + \underset{D}{1} + \underset{E}{2} - 3 \cdot 2 = 1$$

$\boxed{2}$

<u>n° of coordinates</u> : $\{\theta ; \phi ; \varphi ; \psi ; x\}$;

$q = 5$ (absolute coordinates)

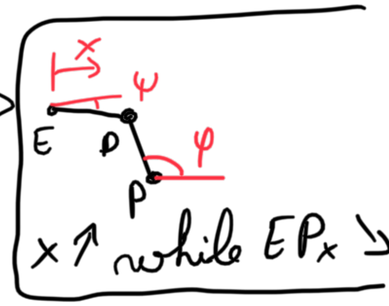
<u>n° of constrains equations</u> (necessary)

$r = q - N = 5 - 1 = 4$

Constrain equations

expressed with respect to the fixed joint P

to join E
$$\begin{cases} x = -ED\cos\psi + DP\cos\varphi \\ EP_y = DP\sin\varphi + ED\sin\psi \end{cases}$$



to joint A
$$\begin{cases} AP_x = AB\cos\theta + BC\cos\phi + CP\cos\varphi \\ AP_y = -AB\sin\theta + BC\sin\phi + CP\sin\varphi \end{cases}$$

EP_x

$\boxed{3}$

Virtual work principle $d\mathcal{T} = \sum_i \bar{R}_i \, dx_i = 0$

$$d\mathcal{T} = F\,dx + C\,d\theta = 0 \iff \boxed{\frac{F}{C} = \frac{-d\theta}{dx}}$$

try to find the time evolution of $(x;\theta)$ using the coordinate partitioning method: $d\Phi = \sum_i \frac{d\Phi}{dq_i} dq_i = 0$

$$\begin{cases} (1)\ dx = ED\sin\psi\, d\psi - DP\sin\varphi\, d\varphi \\ (2)\ 0 = DP\cos\varphi\, d\varphi + ED\cos\psi\, d\psi \\ (3)\ 0 = -AB\sin\theta\, d\theta - BC\sin\phi\, d\phi - CP\sin\varphi\, d\varphi \\ (4)\ 0 = -AB\cos\theta\, d\theta + BC\cos\phi\, d\phi + CP\cos\varphi\, d\varphi \end{cases}$$

$$④ : d\phi = \frac{AB\cos\theta\, d\theta - CP\cos\varphi\, d\varphi}{BC\cos\phi}$$

$$③ : 0 = -AB\sin\theta\, d\theta - \cancel{BC}\, \overset{\text{tg}\,\phi}{\cancel{\sin}\,\phi}\, \frac{AB\cos\theta\, d\theta - CP\cos\varphi\, d\varphi}{\cancel{BC\cos\phi}} - CP\sin\varphi\, d\varphi$$

$$\Leftrightarrow 0 = \left(-AB\sin\theta - AB\,\text{tg}\,\phi\cos\theta\right) d\theta + \left(CP\,\text{tg}\,\phi\cos\varphi - CP\sin\varphi\right) d\varphi$$

$$\Leftrightarrow d\varphi = \frac{\left(AB\sin\theta + AB\,\text{tg}\,\phi\cos\theta\right) d\theta}{CP\,\text{tg}\,\phi\cos\varphi - CP\sin\varphi} = M\, d\theta$$

$$② : \quad d\psi = \frac{-DP\cos\varphi}{ED\cos\psi}\, d\varphi = \frac{-DP\cos\varphi}{ED\cos\psi}\, M\, d\theta$$

$$① : dx = \cancel{ED}\, \overset{\text{tg}\,\psi}{\cancel{\sin}\,\psi}\, \frac{-DP\cos\varphi}{\cancel{ED\cos\psi}}\, M\, d\theta - DP\sin\varphi\, M\, d\theta$$

$$dx = \left(-DP\,\text{tg}\,\psi\cos\varphi - DP\sin\varphi\right) M\, d\theta$$

$$\frac{-d\theta}{dx} = \frac{1}{\left(DP\,\text{tg}\,\psi\cos\varphi + DP\sin\varphi\right) M} = F/C$$