MECA-H303 Vibration TP2

Dynamics of 1 degrees of freedom systems

Pinned bar vibrating

For the following mechanism:

1. Derive the equations of motion of the system using Newtonian dynamics.
2. Derive the equations of motion of the system using Lagrangian dynamics.
3. From the equation of motion, give the expression of the damped and undamped resonant frequency of the system.

Additional details for the exercise:

- Assume small angles of rotation
- Z represents an imposed displacement
- The moment of inertia of the bar is $I = (ml^2)/3$ when computed at its edge.
- The final answer should be expressed as a function of the coordinates on the schematic

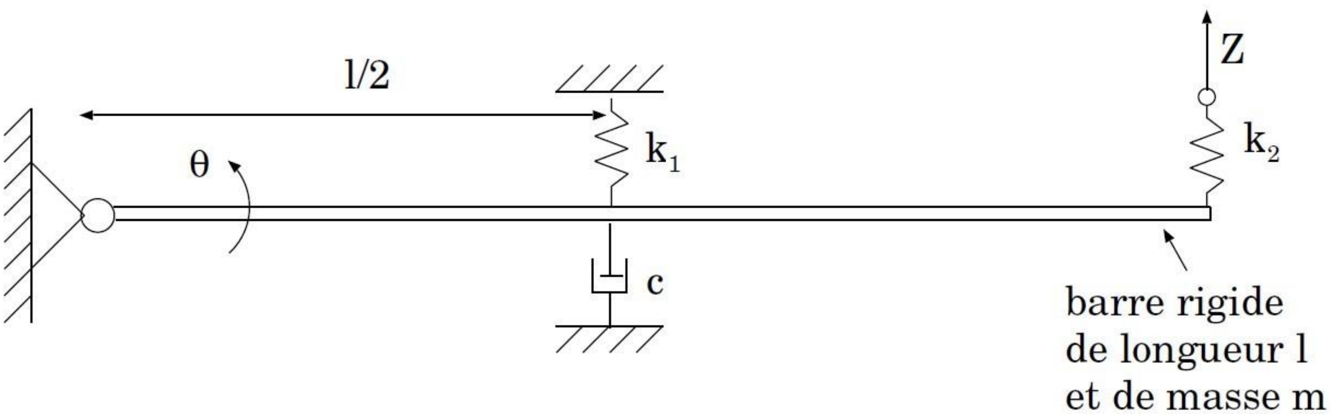


Figure 1: Rigid bar pinned

Shaker on elastic beam

A shaker of mass m_s with two counter-rotating unbalanced masses m_e are mounted on a mass M rigidly attached in the middle of a beam of length L_b.

For this system:

1. Create the equivalent simplified mass-spring-damper system and applied force.
2. Derive the equations of motion of the system using Newtonian dynamics.
3. Derive the magnitude of the response as a funciton of the shaker's excitation frequency.

Additional details for the exercise:

- The stiffness of the beam k can be obtained from the table annexed to the exercise.
- Consider the beam pinned at each edge.

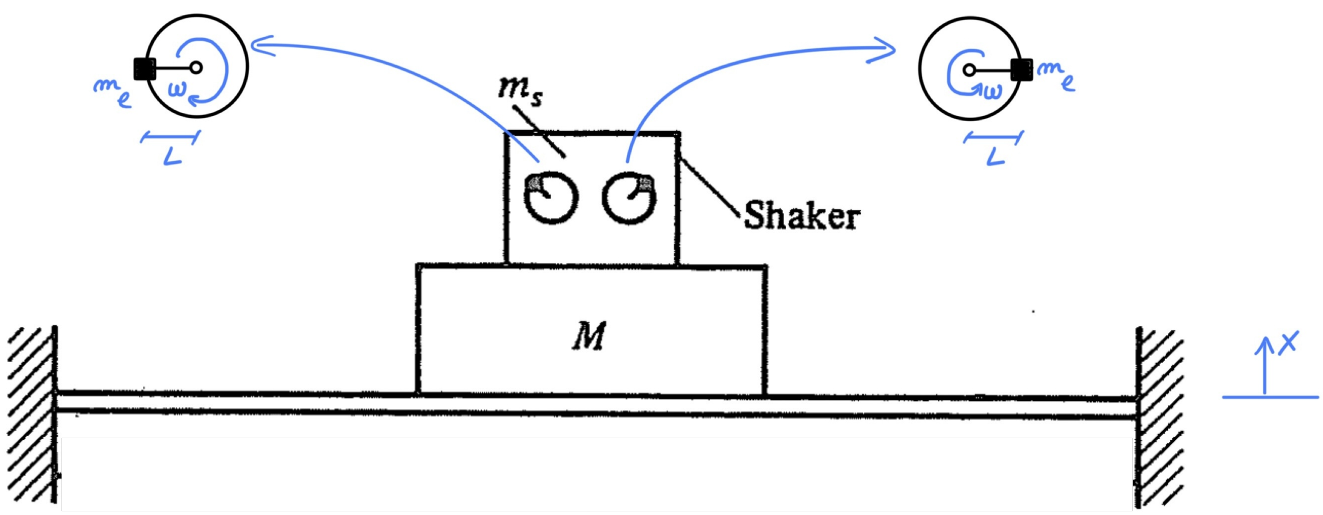


Figure 2: Shaker on elastic beam

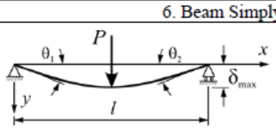
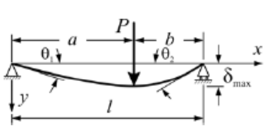
BEAM TYPE	SLOPE AT ENDS	DEFLECTION AT ANY SECTION IN TERMS OF x	MAXIMUM AND CENTER DEFLECTION
6. Beam Simply Supported at Ends – Concentrated load P at the center			
	$\theta_1 = \theta_2 = \frac{Pl^2}{16EI}$	$y = \frac{Px}{12EI}\left(\frac{3l^2}{4} - x^2\right)$ for $0 < x < \frac{l}{2}$	$\delta_{max} = \frac{Pl^3}{48EI}$
7. Beam Simply Supported at Ends – Concentrated load P at any point			
	$\theta_1 = \frac{Pb(l^2 - b^2)}{6lEI}$ $\theta_2 = \frac{Pab(2l - b)}{6lEI}$	$y = \frac{Pbx}{6lEI}\left(l^2 - x^2 - b^2\right)$ for $0 < x < a$ $y = \frac{Pb}{6lEI}\left[\frac{l}{b}(x-a)^3 + \left(l^2 - b^2\right)x - x^3\right]$ for $a < x < l$	$\delta_{max} = \frac{Pb\left(l^2 - b^2\right)^{3/2}}{9\sqrt{3}\,lEI}$ at $x = \sqrt{\left(l^2 - b^2\right)/3}$ $\delta = \frac{Pb}{48EI}\left(3l^2 - 4b^2\right)$ at the center, if $a > b$

Figure 3: Beam rigidity table