Kinematics

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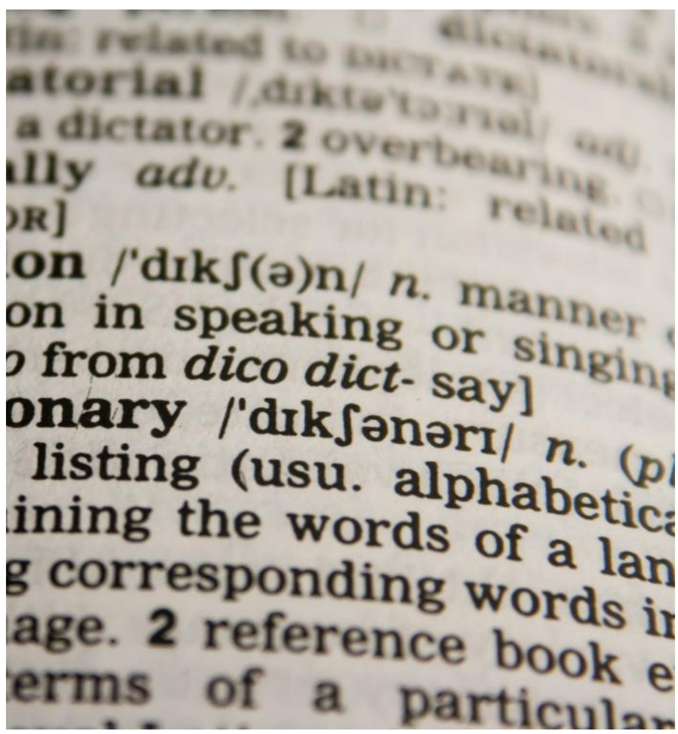
DEFINITIONS



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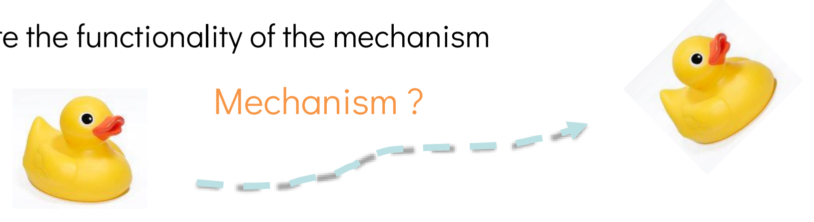
Kinematics

- Kinematics: develop various means of transforming motion to achieve a specific task needed in applications

→ ensure the functionality of the mechanism



- Dynamics: behavior of a given machine or mechanism when subjected to dynamic forces

→ verify the acceptability of induced forces in parts

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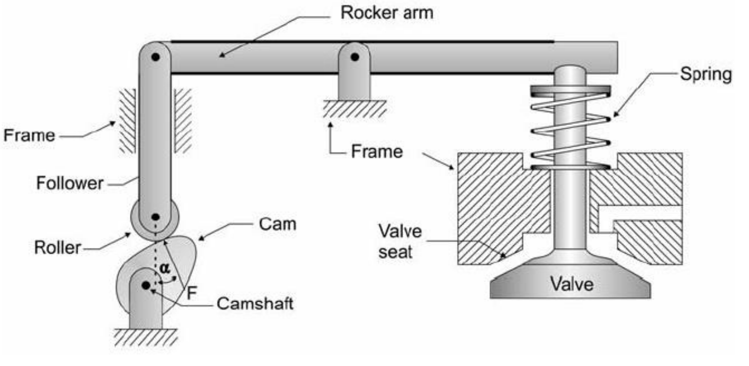
Example : cam operating valve



Kinematical analysis : satisfy functional requirements for valve displacements.

Dynamic analysis : compute forces in the system as a function of time.

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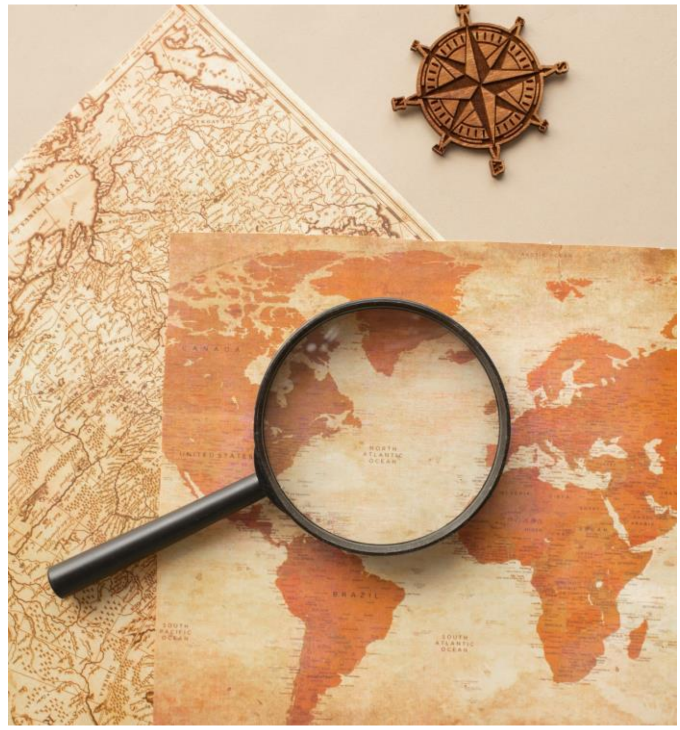
COORDINATES



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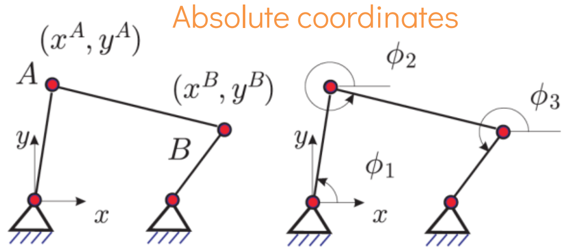
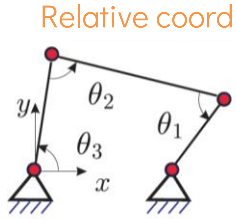
Choice of coordinates

Absolute coordinates



(Cartesian coordinates) (Lagrangian coordinates)

Relative coordinates



- Absolute vs relative
- Choice is not unique
- How many coordinates ?
  (Efficiency vs simplicity)
- How many constraints ?

→ What is the best choice ?

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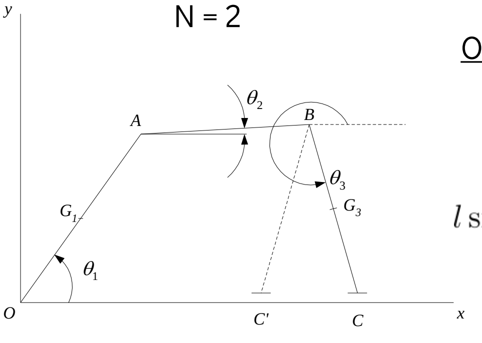
Coordinates and constraints

- N = number of DOFs: minimum number of coordinates required to fully describe the configuration of a system
- Number of coordinates: q ≥ N

⟶ We have to find p=q-N relationships



Option 1: q = 3 : $\theta_1, \theta_2, \theta_3$

⟶ p = 3 – 2 = 1

$$l \sin\theta_3 = -l \sin\theta_1 - l \sin\theta_2 \quad \Big| \quad y_C = 0$$

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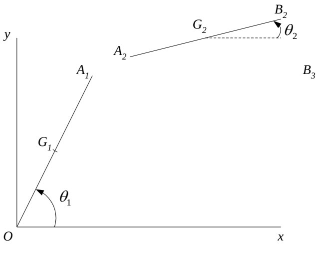
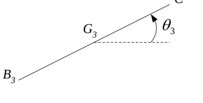
Coordinates and constraints

Option 2: q = 7:

$\theta_1, \theta_2, \theta_3, x_{G2}, y_{G2}, x_{G3}, y_{G3}$ ⟶ p = 7 – 2 = 5

$$1)\; l\cos\theta_1 = x_2 - \frac{l}{2}\cos\theta_2$$
$$2)\; l\sin\theta_1 = y_2 - \frac{l}{2}\sin\theta_2 \quad \Big| \quad A_1 = A_2$$
$$3)\; x_2 + \frac{l}{2}\cos\theta_2 = x_3 - \frac{l}{2}\cos\theta_3$$
$$4)\; y_2 + \frac{l}{2}\sin\theta_2 = y_3 - \frac{l}{2}\sin\theta_3 \quad \Big| \quad B_2 = B_3$$
$$5)\; y_3 + \frac{l}{2}\sin\theta_3 = 0 \quad \Big| \quad y_C = 0$$





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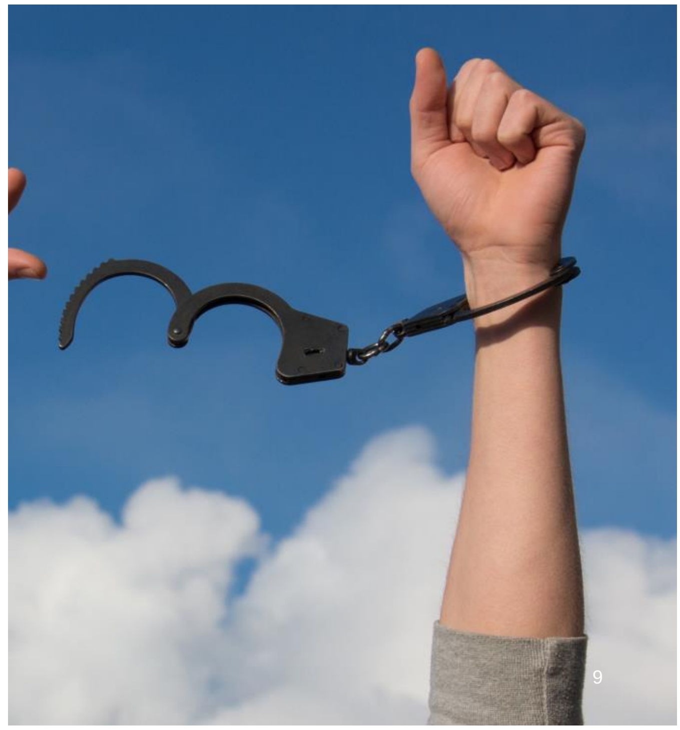
DEGREES OF FREEDOM



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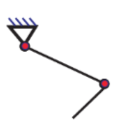
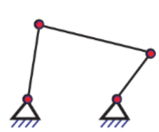
Number of DOFS

- Number of closed loops:
  - *l* number of joints
  - *n* number of bodies

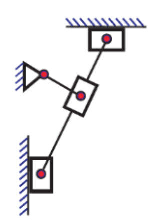
$$b = l - n$$



$b = 0 (l = 2, n = 2)$



$b = 1 (l = 4, n = 3)$



$b = 2 (l = 7, n = 5)$

- Number of DOFs (3D):
  where v_i is the number of DOFS at joint *i*

$$N = \sum_{i=1}^{l} v_i - 6b$$

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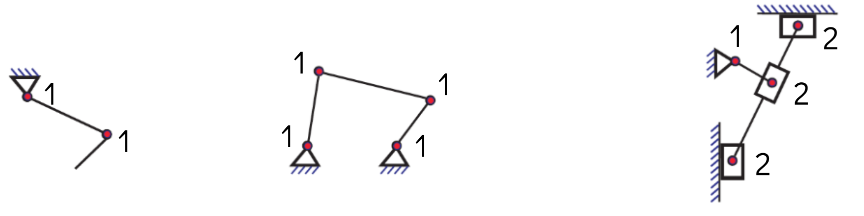
Number of DOFS

- Number of DOFs (2D):
  where v_i is the number of DOFS at joint i

$$N = \sum_{i=1}^{l} v_i - 3b$$



$b = 0(l = 2, n = 2)$
$N = 1 + 1 - 0 = 2$

$b = 1(l = 4, n = 3)$
$N = 4 * 1 - 3 = 1$

$b = 2(l = 7, n = 5)$
$N = 3 * 2 + 1 - 6 = 1$

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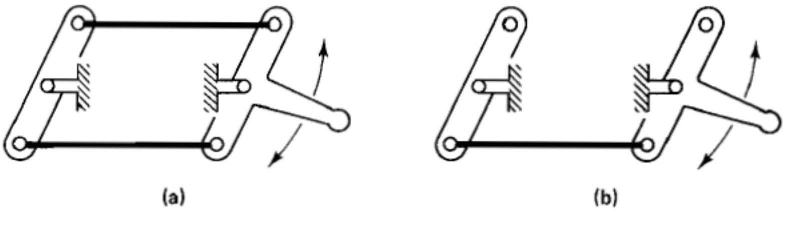
Redundant constraints



A constraint that can be removed without changing the kinematics is called redundant.

→ Not taken into account in the calculation of the number of DOFs.

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CONSTRAINT EQUATIONS FOR PLANAR JOINTS

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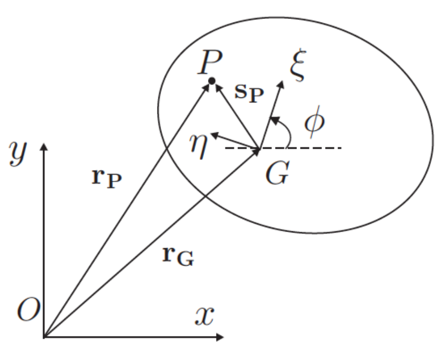
Constraint equations : single body revolute joint



$$\bar{r}_P = \bar{r}_G + A\bar{s}_P$$

$$A = \begin{bmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{bmatrix}$$

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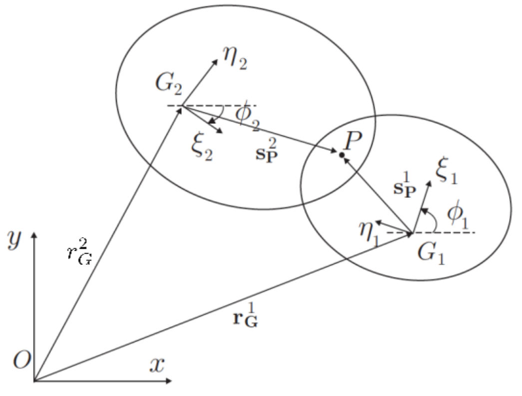
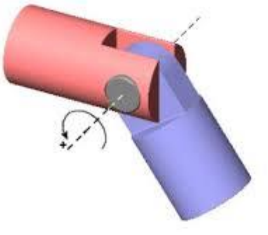
Constraint equations : two bodies revolute joint



$$\overline{r}_G^1 + A_1 \overline{s}_P^1 = \overline{r}_G^2 + A_2 \overline{s}_P^2$$



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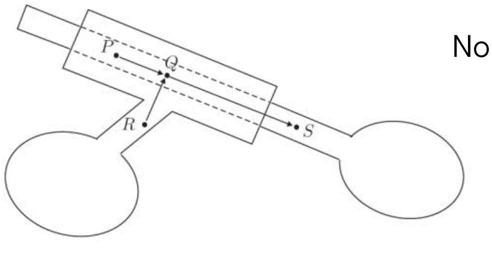
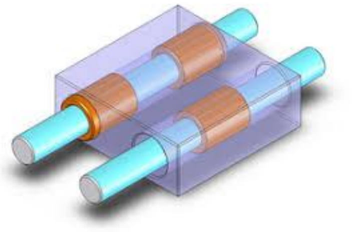
Constraint equations : translation joint





No rotation in the plane between the two bodies

$$\overline{PQ} \times \overline{QS} = 0$$

or $$\overline{RQ} . \overline{QS} = 0$$

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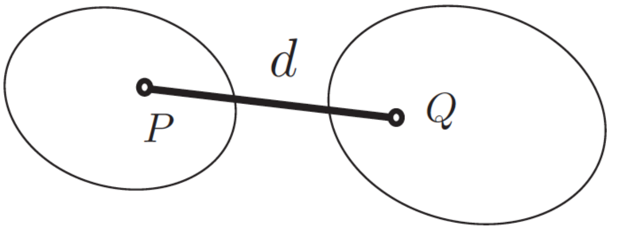
Constraint equations : revolute-revolute joint



Constant distance between P and Q

$$|\overline{PQ}| = d$$

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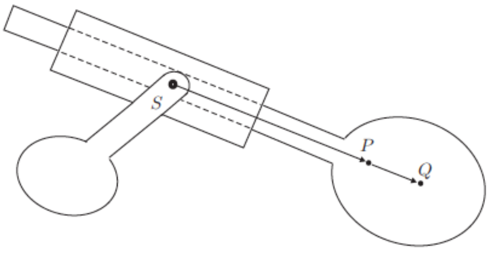
Constraint equations : revolute-translation joint



3 points located on the translation axle remain aligned

$$\overline{SP} \times \overline{PQ} = 0$$

Rotation around S and translation

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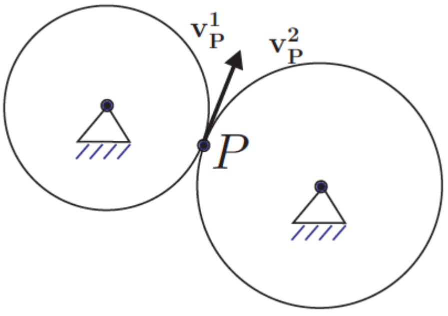
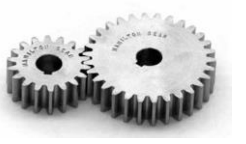
Constraint equations : spur gears





$$\overline{v}_P^1 = \overline{v}_P^2$$
$$\longrightarrow R_1\dot{\theta}_1 = R_2\dot{\theta}_2$$

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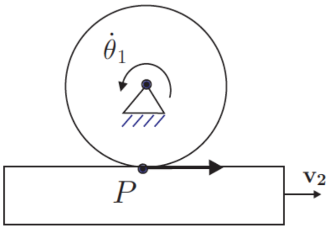
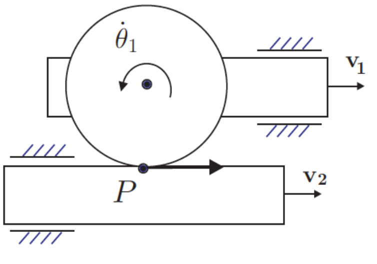
Constraint equations : rack and pinion





$$R_1\dot{\theta}_1 = v_2$$

$$v_1 + R_1\dot{\theta}_1 = v_2$$

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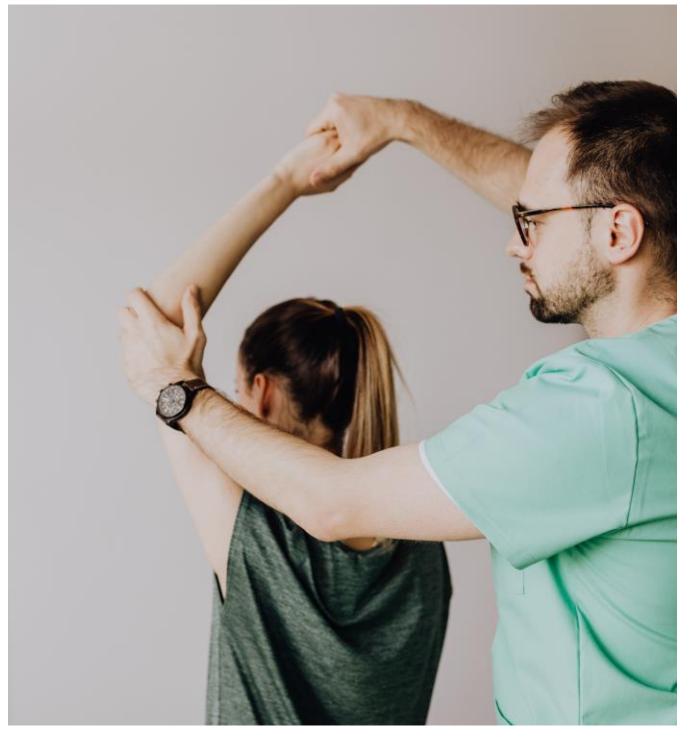
CONSTRAINT EQUATIONS FOR 3D UNIVERSAL JOINT



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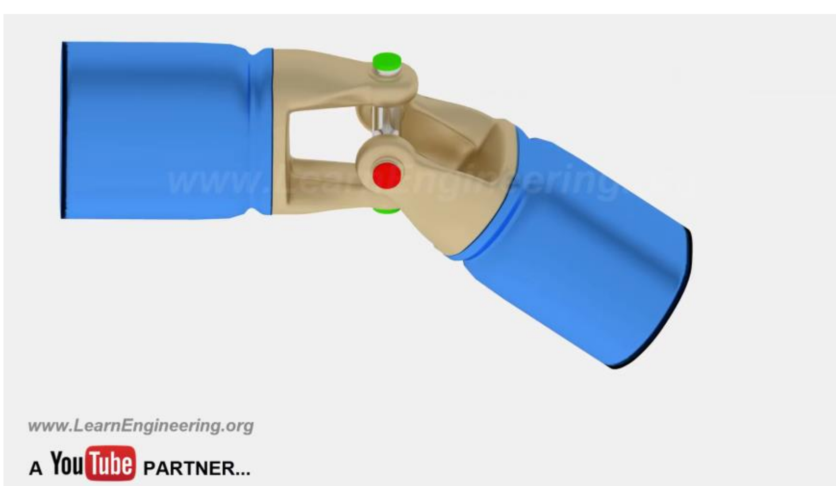
Constraint equations : universal joints



https://www.youtube.com/watch?v=LCMZz6YhbOQ

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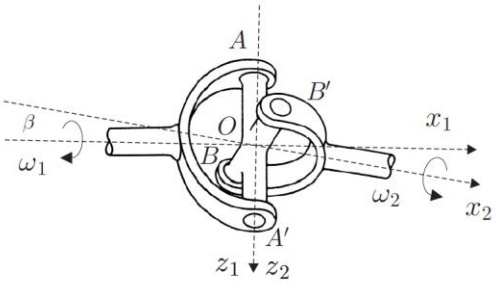
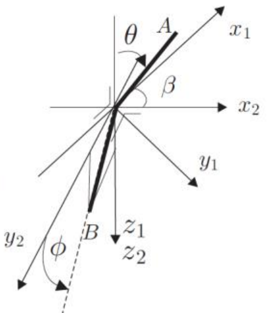
Universal joint equations





$$\overline{OA} = \cos\theta\,\overline{1}_{z_1} + \sin\theta\,\overline{1}_{y_1}$$

$$\overline{OB} = \cos\phi\,\overline{1}_{y_2} + \sin\phi\,\overline{1}_{z_2}$$

$$\overline{OA}\,.\,\overline{OB} = 0 \qquad \overline{1}_{y_1}\,.\,\overline{1}_{y_2} = \cos\beta \qquad \overline{1}_{z_1}\,.\,\overline{1}_{y_2} = 0 \qquad \overline{1}_{z_2}\,.\,\overline{1}_{y_1} = 0$$

$$-\cos\theta\sin\phi + \cos\beta\sin\theta\cos\phi = 0 \longrightarrow \tan\phi = \cos\beta\tan\theta$$

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Universal joint equations

$\tan\phi = \cos\beta\tan\theta$ Differentiation: $\dfrac{1}{\cos^2\phi}\dot{\phi} = \dfrac{\cos\beta}{\cos^2\theta}\dot{\theta}$

$$\frac{1}{\cos^2\phi} = 1 + \tan^2\phi = 1 + \cos^2\beta\tan^2\theta$$

$$\boxed{\omega_2 = \frac{\omega_1\cos\beta}{1 - \sin^2\beta\sin^2\theta}}$$

$\beta <<$

Transmission is not uniform unless the two axles are aligned

$$\dot{\omega}_2 = \frac{\omega_1^2\sin^2\beta\cos\beta\sin 2\theta}{\left(1 - \sin^2\beta\sin^2\theta\right)^2}$$

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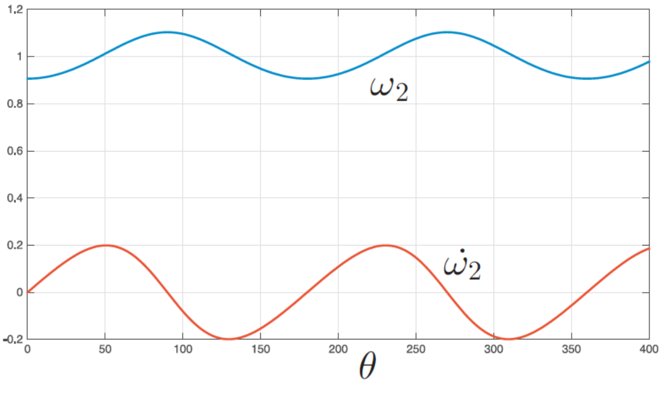
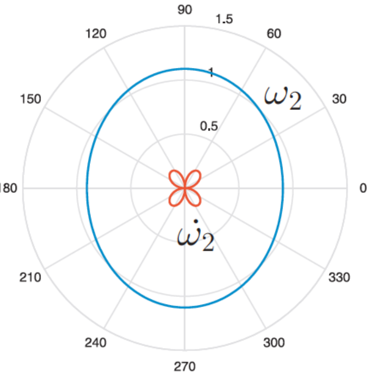
Universal joint equations

$\beta = 25^\circ$ → 10% variation of ω_2





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