Newtonian Dynamics

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NEWTONIAN's SECOND LAW FOR A SOLID

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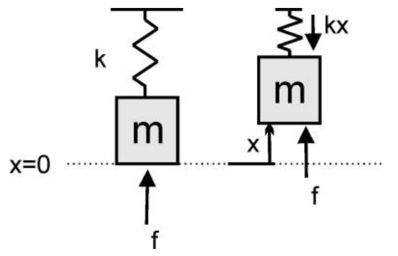
Linear momentum – Mass spring system

Linear momentum (translation)

$$m\frac{d\overline{v}_G}{dt} = \sum_{i=1}^{N} \overline{F}_{ext,h}$$



- Spring force: $-kx$
- External force f acting on the mass.

$$m\ddot{x} = \sum F_x \longrightarrow m\ddot{x} + kx = f$$

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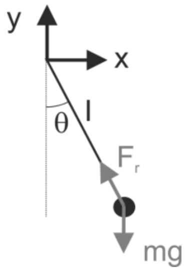
Pendulum



Linear momentum (translation)

$$m\ddot{x} = -F_r \sin\theta$$
$$m\ddot{y} = F_r \cos\theta - mg$$

Accelerations

$$x = l\sin\theta \quad \dot{x} = l\dot{\theta}\cos\theta \longrightarrow \ddot{x} = l\ddot{\theta}\cos\theta - l\dot{\theta}^2\sin\theta$$
$$y = -l\cos\theta \quad \dot{y} = l\dot{\theta}\sin\theta \qquad \ddot{y} = l\ddot{\theta}\sin\theta + l\dot{\theta}^2\cos\theta$$

$$F_r = \frac{-m\ddot{x}}{\sin\theta}$$
$$m\ddot{y} = -m\ddot{x}\frac{\cos\theta}{\sin\theta} - mg \longrightarrow ml\ddot{\theta} + mg\sin\theta = 0$$

Small displacements

$$\longrightarrow ml\ddot{\theta} + mg\theta = 0$$

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Angular momentum

Angular momentum (rotation)

$$\frac{d\overline{M}_A}{dt} = m\overline{v}_G \times \overline{v}_A + \overline{m}_{ext,A}$$

- $G = A$
- $\overline{v}_G = 0$
- $\overline{v}_A = 0$
- $\overline{v}_G \parallel \overline{v}_A$

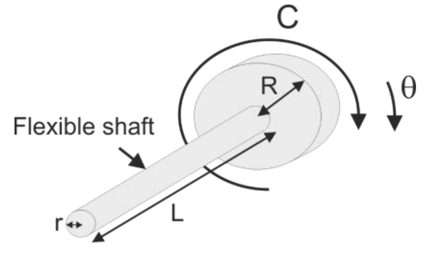
$\rightarrow$

$$\frac{d\overline{M}_A}{dt} = \overline{m}_{ext,A}$$

At the center of gravity

$$\frac{d\overline{M}_G}{dt} = \overline{m}_{ext,G} \qquad \overline{M}_G = \overline{\overline{I}}_G \,.\, \overline{\omega}$$

$$\overline{M}_G = I\dot{\theta} \longrightarrow I\ddot{\theta} + K\theta = C$$



$$I = \frac{1}{4}MR^2 \qquad K = \frac{E\pi r^4}{4(1+\nu)L}$$

Rotational spring

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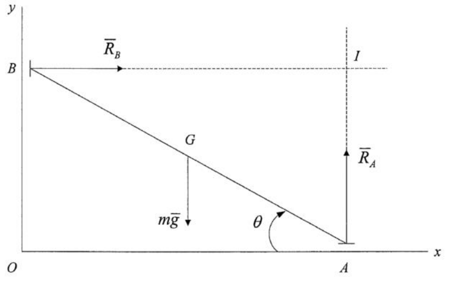
Sliding bar

$$m\frac{d\overline{v}_G}{dt} = \sum_{i=1}^{N} \overline{F}_{ext,h}$$



$$\overline{r}_G = \frac{L}{2}(\cos\theta\overline{1}_x + \sin\theta\overline{1}_y)$$

$$\overline{v}_G = \frac{L\dot{\theta}}{2}(-\sin\theta\overline{1}_x + \cos\theta\overline{1}_y)$$

$$\overline{a}_G = -\frac{L}{2}\left((-\ddot{\theta}\sin\theta - \dot{\theta}^2\cos\theta)\overline{1}_x + (\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta)\overline{1}_y\right)$$

$$\frac{mL}{2}(-\ddot{\theta}\sin\theta - \dot{\theta}^2\cos\theta) = \underline{R_B}$$

$$\frac{mL}{2}(\ddot{\theta}\cos\theta - \dot{\theta}^2\sin\theta) = \underline{R_A} - mg$$

Reaction forces

2 equations/
3 unknowns

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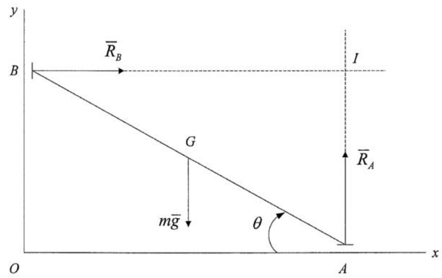
Sliding bar

$$\frac{\mathrm{d}\overline{M}_I}{\mathrm{d}t} = mg\frac{L}{2}\cos\theta \overline{1}_z \qquad \overline{v}_G \parallel \overline{v}_I$$



$$\overline{M}_I = \overline{M}_G + m\overline{IG} \times \overline{v}_G = -\frac{mL^2}{3}\dot{\theta}\overline{1}_z$$

$$\overline{M}_G = \frac{mL^2}{12}$$

Choice of I allows to get rid of reaction forces R_A and R_B

$$\rightarrow \quad -\frac{mL^2}{3}\ddot{\theta} = mg\frac{L}{2}\cos\theta$$

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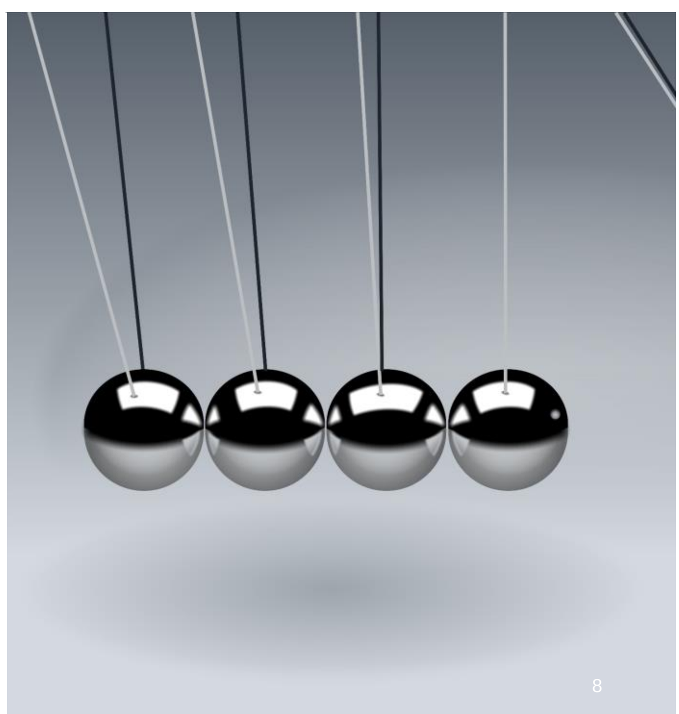
NEWTONIAN's LAWS FOR MULTIPLE SOLIDS



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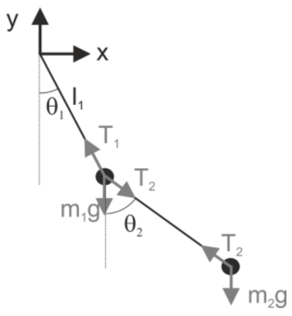
Double pendulum



Linear momentum (translation)

$$m_1\ddot{x}_1 = -T_1 \sin\theta_1 + T_2 \sin\theta_2$$
$$m_1\ddot{y}_1 = T_1 \cos\theta_1 - T_2 \cos\theta_2 - m_1 g$$

$$m_2\ddot{x}_2 = -T_2 \sin\theta_2$$
$$m_2\ddot{y}_2 = T_2 \cos\theta_2 - m_2 g$$

Accelerations

$$x_1 = l_1 \sin\theta_1 \qquad y_1 = -l_1 \cos\theta_1$$
$$x_2 = l_1 \sin\theta_1 + l_2 \sin\theta_2 \qquad y_2 = -l_1 \cos\theta_1 - l_2 \cos\theta_2$$

Compute $\dot{x}_1, \ddot{x}_1, \dot{x}_2, \ddot{x}_2$ And replace in equilibrium equations ....

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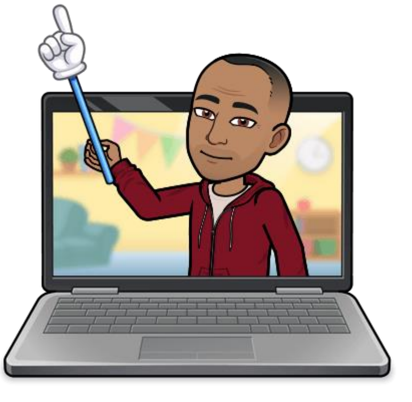
Conclusion

- Newton's second law must be applied to each solid separately
- It introduces (unknown) reaction forces
- For multiple solids, it generally leads to lengthy calculations
- The use of Lagrange equations is an alternative.



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