Statics

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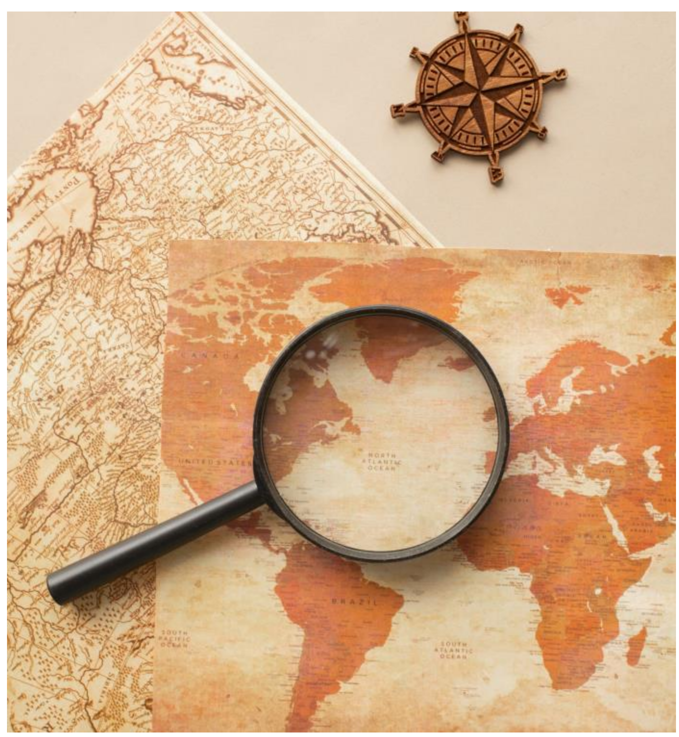
COORDINATES



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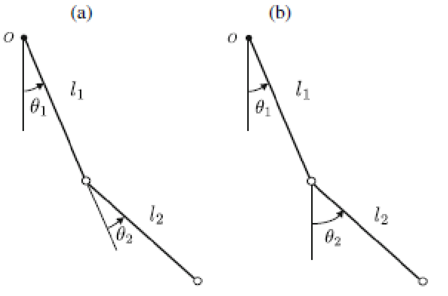
Generalized coordinates

Degrees of freedom N : minimum number of coordinates necessary to provide the full geometric description of the system



Generalized coordinates : $q_1, q_2, \dots q_n$

Minimum set of coordinates: $n = N$

$n > N$ → Need for kinematic constraints

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Kinematic constraints

$f(q_1, \dots, q_n, t) = 0$ Holonomic constraints

$f(q_1, \dots, q_n) = 0$ Scleronomic constraints

→ Possibility to reduce to a minimum set

$\sum_i a_i dq_i + a_0 dt = 0$ or $\sum_i a_i dq_i = 0$ Non holonomic constraints

→ Not possible to reduce to a minimum set

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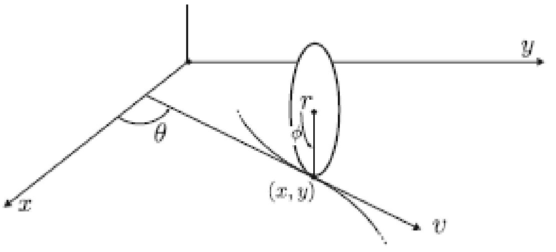
Example of non holonomic constraints



- 4 generalized coordinates

  x, y, θ, ϕ

- All positions and orientations of the disk are possible

But, time derivatives of coordinates are not independent

$$v = r\dot{\phi}$$
$$\dot{x} = v\cos\theta$$
$$\dot{y} = v\sin\theta$$

$$dx - r\cos\theta d\phi = 0$$
$$dy - r\sin\theta d\phi = 0$$

Not all paths are possible to go from one configuration to another

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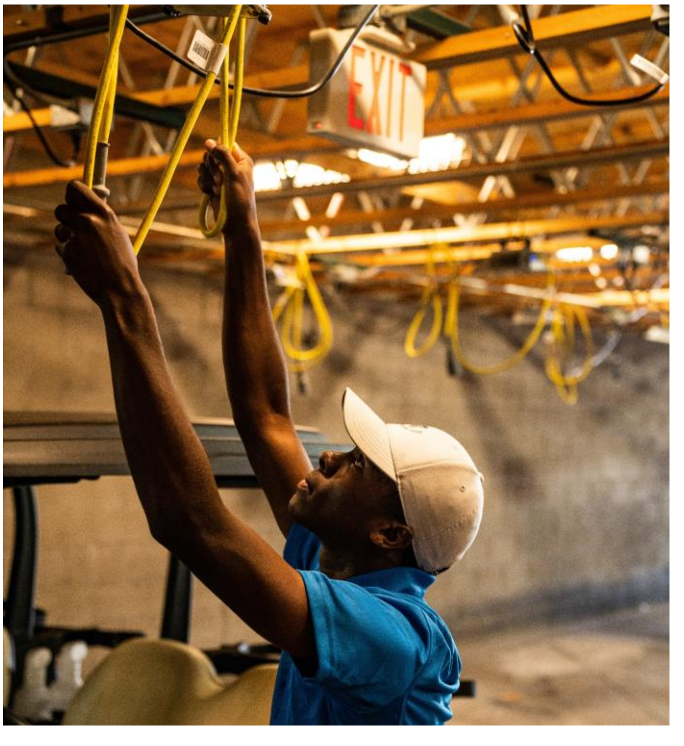
VIRTUAL DISPLACEMENTS and VIRTUAL WORK



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Virtual displacements

Infenitesimal change of coordinates

- Occuring at a constant time
- Compatible with the kinematic constraints

δq_i Virtual change of coordinates

$$\delta f = \sum_i \frac{\partial f}{\partial q_i} \delta q_i = 0$$

For holonomic constraints

Virtual displacements

For a particle constrained to a frictionless surface

$$f(x, y, z) = 0 \longrightarrow \frac{\partial f}{\partial x}\delta x + \frac{\partial f}{\partial y}\delta y + \frac{\partial f}{\partial z}\delta z = 0$$

$$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)^T$$

is parallel to the normal to the surface

$$\overline{\delta x} = (\delta_x, \delta_y, \delta_z)^T \longrightarrow \nabla f \cdot \overline{\delta x} = (\nabla f)^T \overline{\delta x} = 0$$

Virtual displacements

The reaction force vector $\overline{F'}$ which constraints the particle to the surface is normal to the surface (no friction)

$$\overline{F'} \cdot \overline{\delta x} = \overline{F'}^T \overline{\delta x} = 0$$

→ The virtual work of the constraint forces is zero for any virtual displacement

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Principle of virtual work

Consider a set of N particles with position vectors $\overline{x}_i,\ i = 1, ..., N$ and the resultant force on each of these particles is $\overline{R}_i$

Static equilibrium

$$\sum_{i=1}^{N} \overline{R}_i \cdot \overline{\delta x}_i = 0$$

since $\overline{R}_i = 0$ at equilibrium

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Principle of virtual work

Static equilibrium

$$\sum_{i=1}^{N} \overline{R}_i \cdot \overline{\delta x}_i = 0$$

$$\overline{R}_i = \overline{F}_i + \overline{F}_i'$$

Applied forces

Reaction forces

The virtual work of the constraint forces is zero for any virtual displacement

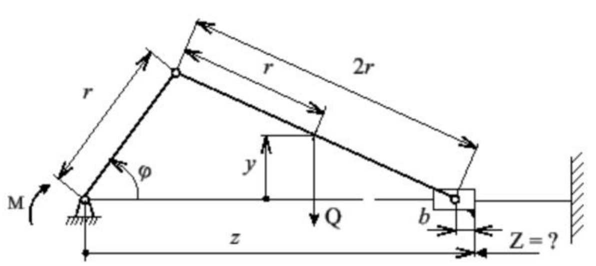
$$\overline{F}_i' \cdot \overline{\delta x}_i = 0$$

$$\sum_{i=1}^{N} \overline{F}_i \cdot \overline{\delta x}_i = 0$$

Virtual work of external forces is zero

Slider-crank

Problem: Find *Z* to maintain the slider-crank mechanism in static equilibrium with ϕ = 30°.



M = 50 Nm
Q = 35 N
r = 0.1 m

Newtonian mechanics :

2 equations for translation
1 equation for rotation

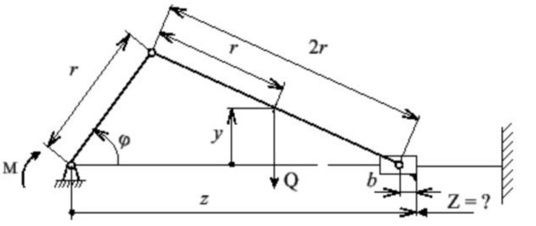
X 2 bodies = 6 equations

→ 5 reaction forces + Z = 6 unknowns

Slider-crank

Forces: Q, Z;   Moment: M

$$q_1 = \varphi$$



Virtual work:

$$-M\delta\varphi - Q\delta y - Z\delta z$$

$$y = \frac{r}{2}\sin\varphi \longrightarrow \delta y = \frac{r}{2}\cos\varphi\delta\varphi$$

$$z = r\cos\varphi + r\sqrt{4-\sin^2\varphi} + b \longrightarrow \delta z = -r\sin\varphi\left(1 + \frac{\cos\varphi}{\sqrt{4-\sin^2\varphi}}\right)\delta\varphi$$

$$-M\delta\varphi - Q\delta y - Z\delta z = 0$$

$$\varphi = 30^\circ \longrightarrow Z = 711.92\ N$$

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