Tuned Vibration Absorbers

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UNDAMPED TUNED ABSORBERS

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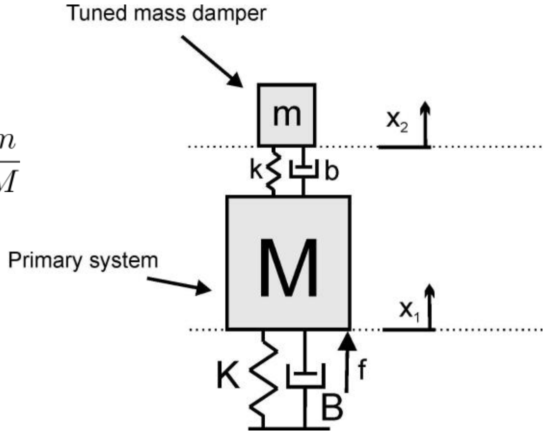
Tuned mass damper principle

$$\mu = \frac{m}{M}$$



$$\omega_n = \sqrt{\frac{k}{m}}$$

$$\Omega = \sqrt{\frac{K}{M}}$$

$$\nu = \frac{\omega_n}{\Omega}$$

Equations of motion:

$$\begin{bmatrix} M & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} B+b & -b \\ -b & b \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} \begin{bmatrix} K+k & -k \\ -k & k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} f \\ 0 \end{Bmatrix}$$

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TMD equations

Harmonic excitation:

$$\begin{bmatrix} K+k+i\omega(B+b)-\omega^2 M & -(k+i\omega b) \\ -(k+i\omega b) & k-\omega^2 m + i\omega b \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix}$$

$$X_1/F = \frac{k-\omega^2 m + i\omega b}{(K+k+i\omega(B+b)-\omega^2 M)(k-\omega^2 m+i\omega b)-(k+i\omega b)^2}$$

Undamped vibration absorber (b=0)

$$X_1/F = \frac{k-\omega^2 m}{(K+k+i\omega B-\omega^2 M)(k-\omega^2 m)-k^2}$$

$$X_1 = 0 \quad \text{for} \quad \omega = \sqrt{\frac{k}{m}} = \omega_n$$

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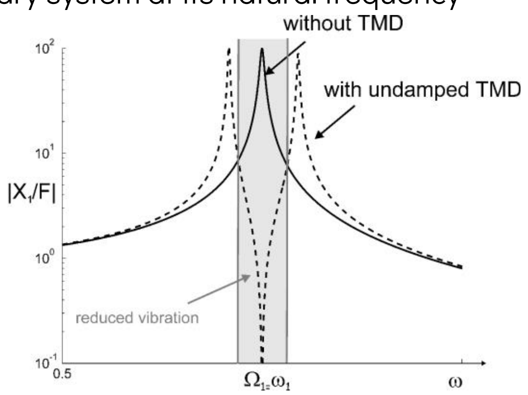
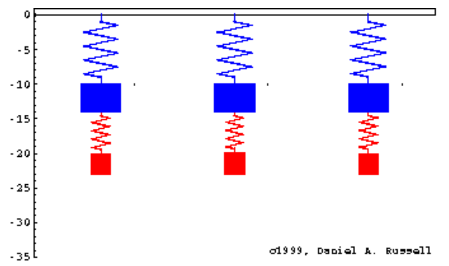
Undamped TMD

If you choose $\omega_n = \Omega$ you can cancel the vibration of the primary system at its natural frequency

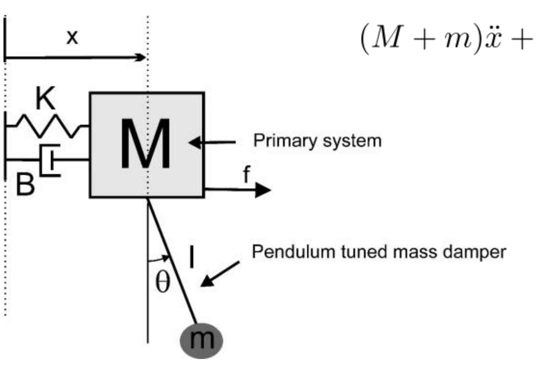


$\mu = 0.03,\ \nu = 1$



The damping device is tuned to the eigenfrequency of the primary system
-> Reduces vibrations in a narrow band around eigenfrequency
-> Amplification outside of this narrow band

Pendulum TMD



$$
\begin{aligned}
(M+m)\ddot{x} + ml\ddot{\theta} + Kx + B\dot{x} &= f \\
m(\ddot{x} + l\ddot{\theta}) + mg\theta &= 0
\end{aligned}
$$

θ small

$$\omega_n = \sqrt{\frac{g}{l}} \qquad \Omega = \sqrt{\frac{K}{M}}$$

$$\nu = \frac{\omega_n}{\Omega} \qquad \mu = \frac{m}{M}$$

$$
\begin{bmatrix} M+m & ml \\ 1 & l \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} K & 0 \\ 0 & g \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} f \\ 0 \end{Bmatrix}
$$

Inertial coupling of the two systems

PTMD equations

Harmonic excitation:

$$\begin{bmatrix} K + i\omega B - \omega^2(M+m) & -ml\omega^2 \\ -\omega^2 & g - \omega^2 l \end{bmatrix} \begin{Bmatrix} X \\ \Theta \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix}$$

$$\frac{X}{F} = \frac{g - \omega^2 l}{(K + i\omega B - \omega^2(M+m))(g - \omega^2 l) + \omega^4 ml}$$

$$X = 0 \quad \text{for} \quad \omega = \sqrt{\frac{g}{l}} = \omega_n$$

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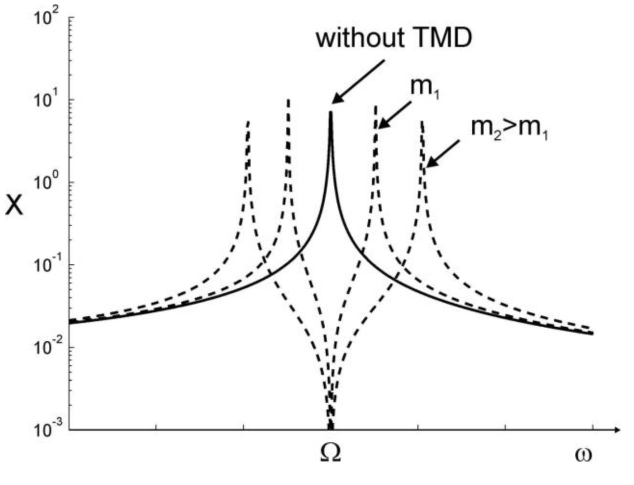
Undamped PTMD

Tuning of the PTMD based on the length of the pendulum $\omega_n = \sqrt{\frac{g}{l}}$



$\nu = 1$

Effect of the mass mainly on the spreading of the peaks

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DAMPED TUNED ABSORBERS

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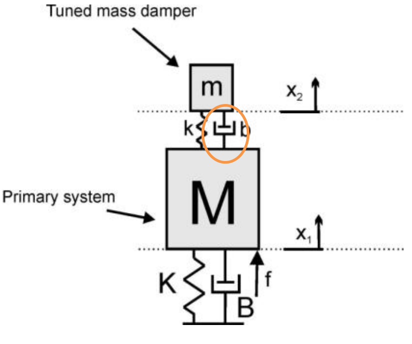
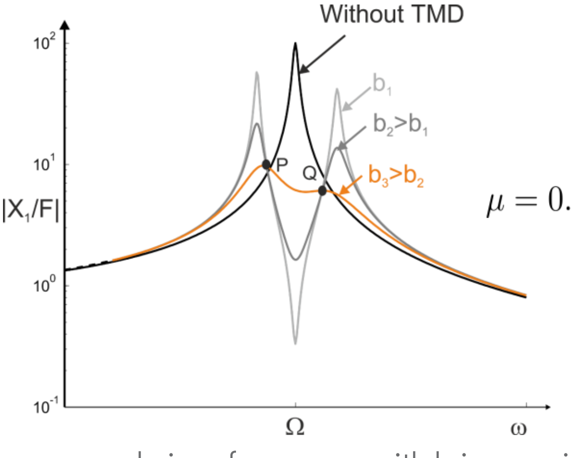
Damped TMD





$$\mu = 0.03,\ \nu = 1$$

-Reduction of vibration is lower around eigenfrequency with b increasing
-Reduces the amplification outside of the narrow frequency band
-Existence of P and Q : points where all curves cross

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Optimal design of TMD

P and Q are at equal height for

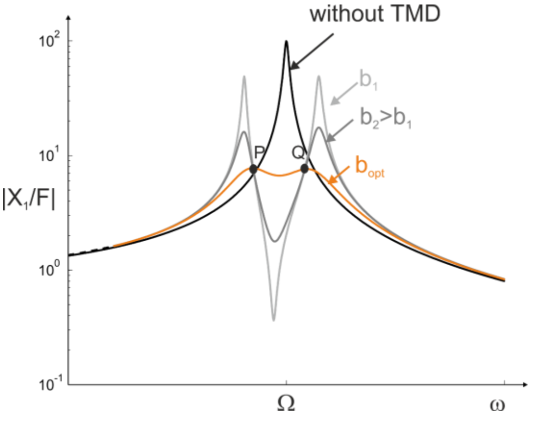
$$\nu = \frac{1}{1+\mu}$$

Optimum damping is given by

$$\xi = \sqrt{\frac{3\mu}{8(1+\mu)}}$$

$\longrightarrow$ $b = 2\xi\sqrt{km}$

[Den Hartog, 1954]



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Optimal design of TMD

- The maximum mass of the device is decided fixing $\mu = \frac{m}{M}$

- Based on this value, the frequency of the TMD is tuned : $\nu = \frac{1}{1+\mu}$

- Which allows to compute the stiffnes of the TMD

$$k = \nu^2 K\mu = K\frac{\mu}{(1+\mu)^2}$$

- And finally the optimal damping is computed

$$\xi = \sqrt{\frac{3\mu}{8(1+\mu)}} \longrightarrow b = 2\xi\sqrt{km}$$

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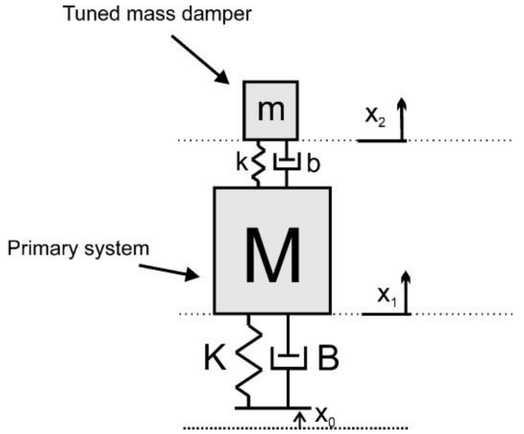
Optimal design of TMD

Goal : minimize $|\frac{X_1 - X0}{-\omega^2 X_0}|$



$$\nu = \frac{\sqrt{1-\mu/2}}{1+\mu}$$

$$\xi = \sqrt{\frac{3\mu}{8(1+\mu)(1-\mu/2)}}$$

[Warburton 1982]

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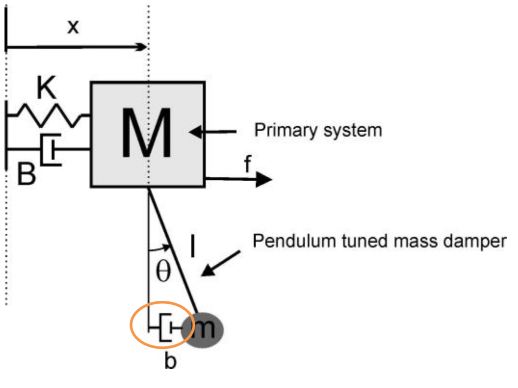
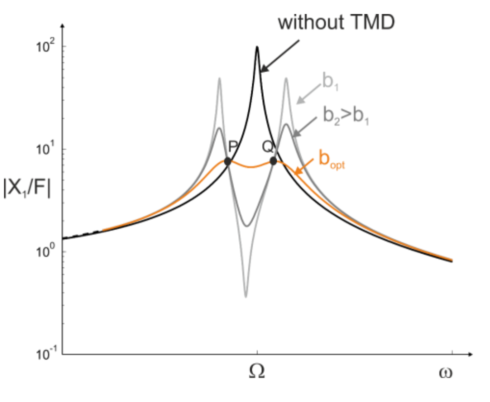
Damped PTMD





Analogy with TMD
[Deraemaeker 2020]

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APPLICATIONS IN MECHANICAL ENGINEERING

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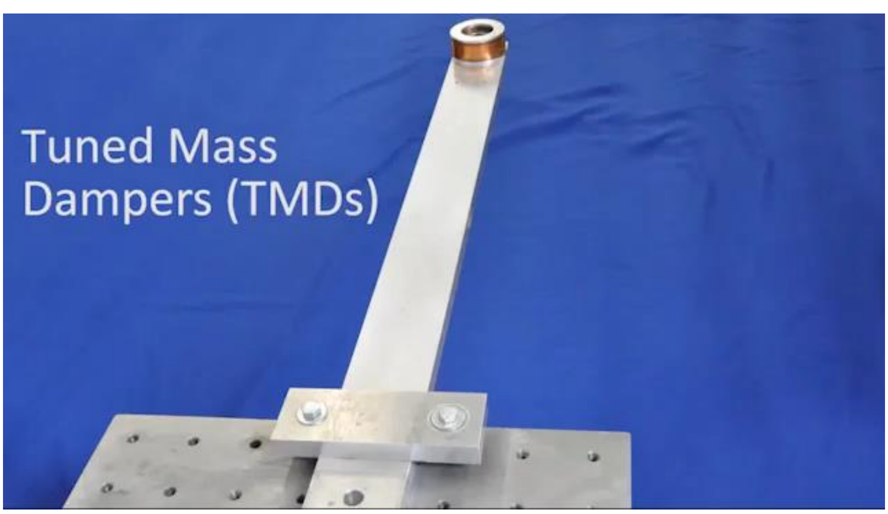
Example : beam equipped with a TMD



https://youtu.be/HDa1VO1VDpc

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TMD for pipe vibrations

https://www.youtube.com/watch?v=X25DJ1_po8s

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APPLICATIONS IN CIVIL ENGINEERING

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Example : building equipped with a TMD

https://youtu.be/lhNjfNUOUo8

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Pendulum TMD

https://youtu.be/GzMuF-LMGaM

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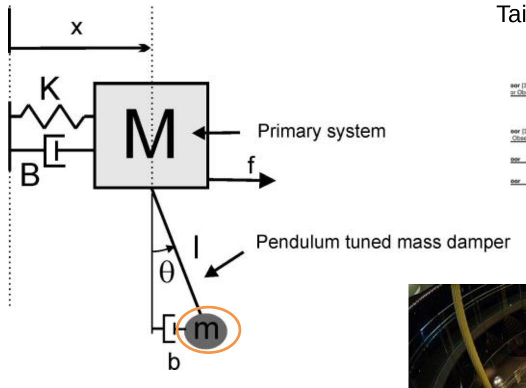
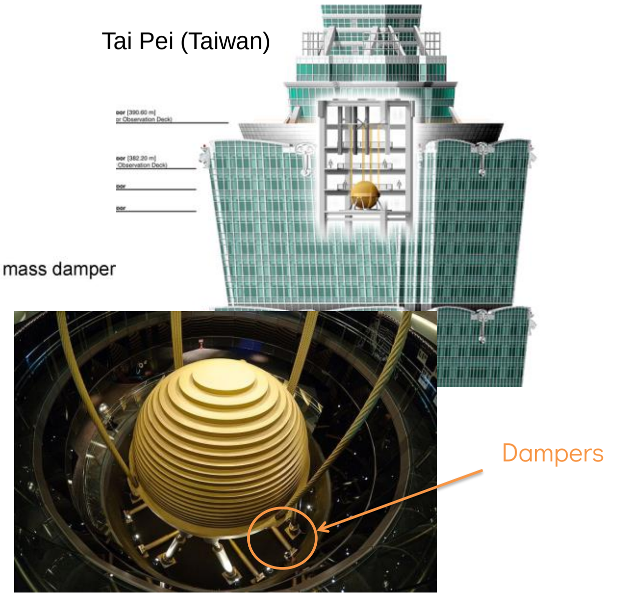
Pendulum TMD in high-rise buildings





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