MECA-H303 TP1

Kinematics & Statics

Simple lift mechanism

For the following mechanism, we ask you to:

1. Calculate the number of degrees of freedom
2. Write the constraint equations
3. Using the principle of virtual work, express the ratio between F_x and F_y as a function of the system's coordinates, when the system is in static equilibrium.

Additional details for the exercise:

- Use the coordinate system centered on point C
- Lengths $|AE| = |BE| = |CE| = |DE| = L$
- The mass of all bars is negligible

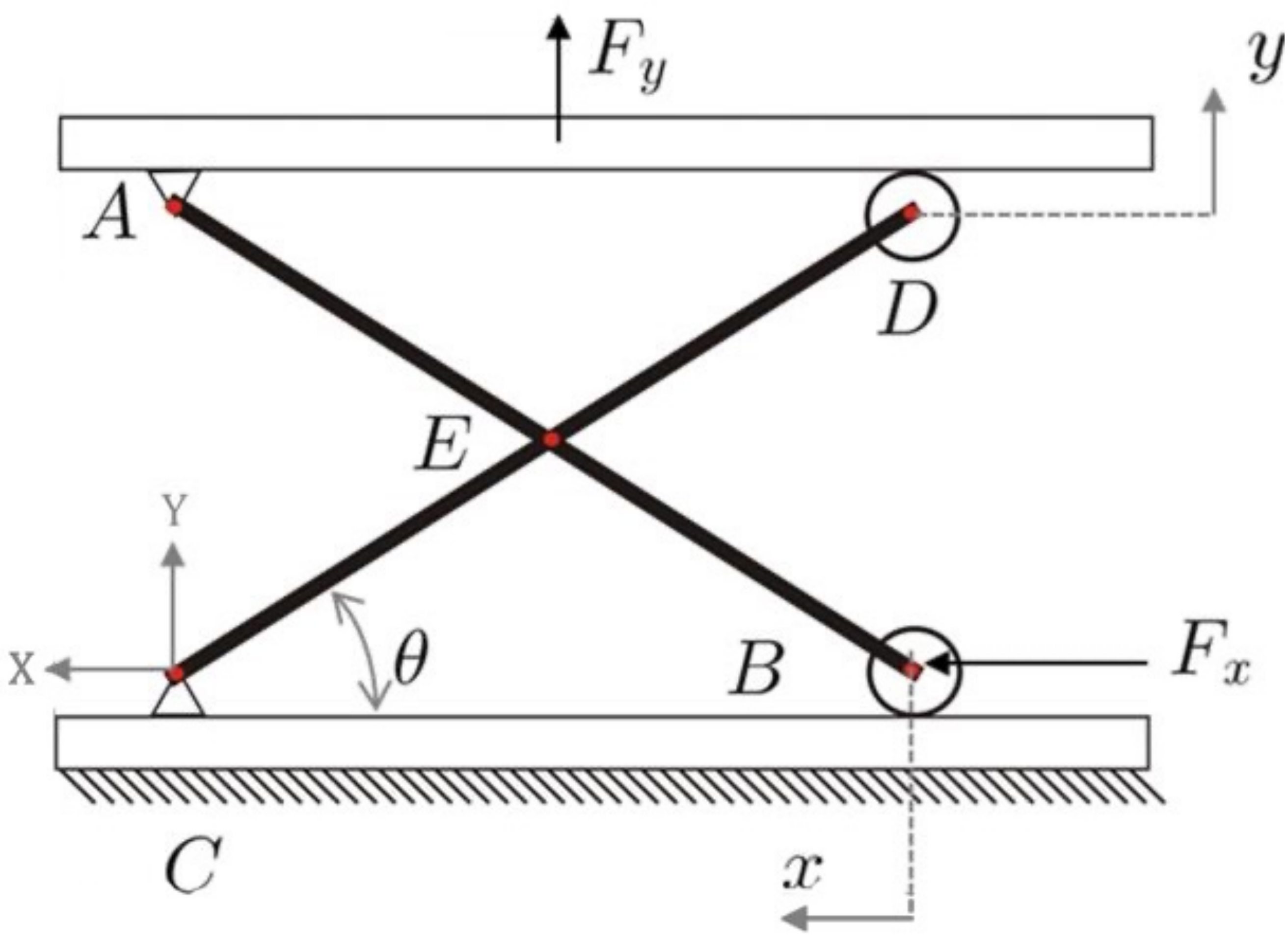


Figure 1: simple lift mechanism

1

$$N = \left(\sum_{i=1}^{l} v_i\right) - 3b$$

m : n° of bodies

l : n° of joints

v_i : n° of dofs at joint i

$b = l - m = 5 - 3 = 2$ (n° of closed loops)

$$N = 1 + 2 + 1 + 2 + 1 - 3.2 = 1 \quad \text{(n° of dofs)}$$

joint: A B C D E

2

$$x_B = -2L\cos\theta$$

$$y_D = 2L\sin\theta$$

3

$$\delta\bar{\tau} = 0$$

$$\Leftrightarrow \delta\bar{F}_x + \delta\bar{F}_y = 0$$

$$\Leftrightarrow F_x\, dx_B + F_y\, dy_A = 0$$

$$\begin{cases} dx_B = 2L\, d\theta \sin\theta \\ dy_D = 2L\, d\theta \cos\theta \end{cases}$$

$$\Leftrightarrow \frac{F_y}{F_x} = \frac{-dx_B}{dy_D} = \frac{-\sin\theta}{\cos\theta} = -tg\,\theta$$

F_x and F_y need to be of opposite signs to have the lifting plate in static equilibrium

Crank and slider

For the following mechanism, we ask you to:

1. Calculate the number of degrees of freedom
2. Write the constraint equations
3. Using the principle of virtual work, express the ratio between the force F and the torque C applied as a function of the system's coordinates, when the system is in static equilibrium.

Additional details for the exercise:

- DC is one rigid bar pinned in point P

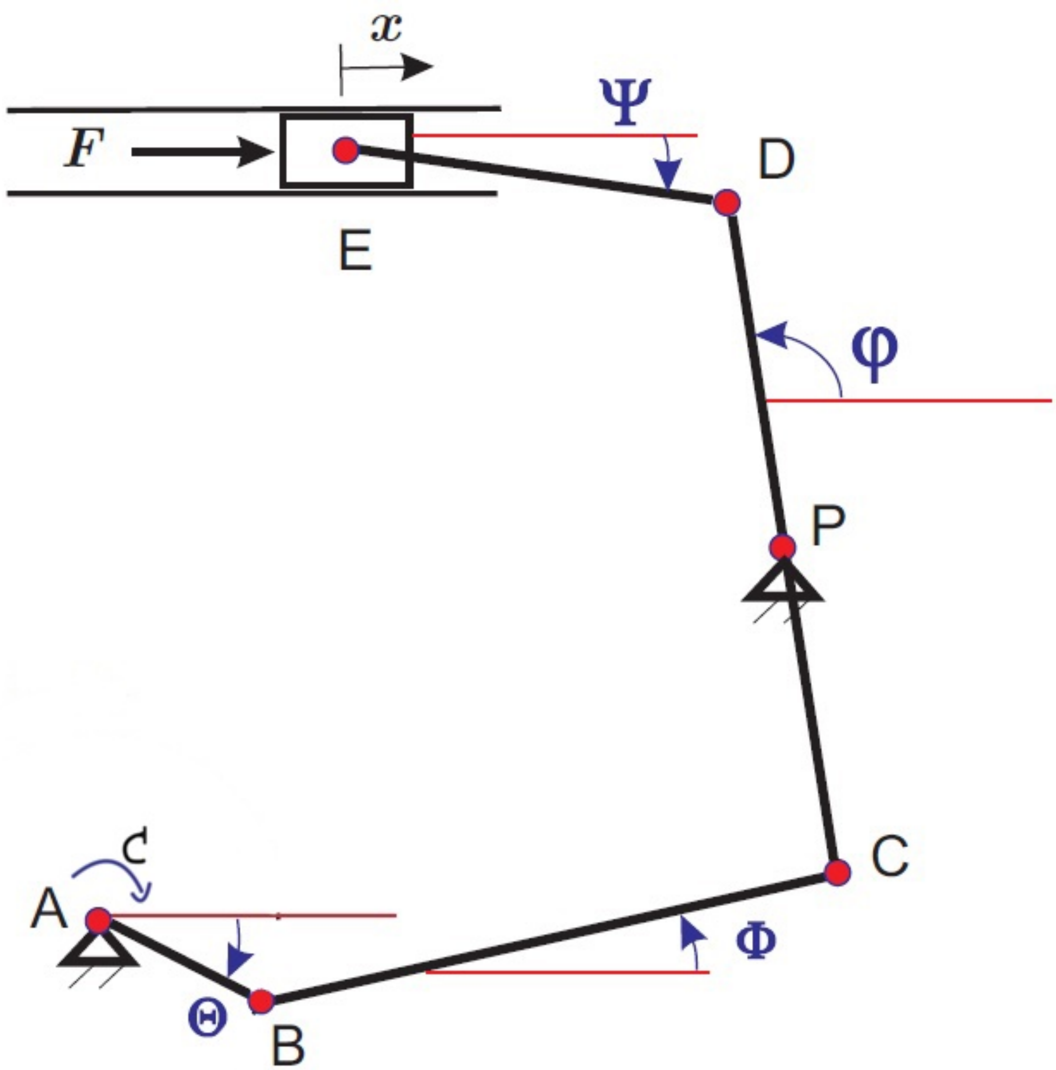


Figure 2: Crank and slider

1

$$N = \sum_{i=1}^{l} v_i - 3b$$

v_i : n° of dofs at joint i

b : n° of closed loops

$= l - n = 6 - 3 = 2$

n : n° of bodies

l : n° of joints

$$N = \underset{A}{1} + \underset{B}{1} + \underset{C}{1} + \underset{P}{1} + \underset{D}{1} + \underset{E}{2} - 3 \cdot 2 = 1$$

2

n° of coordinates : $\{\theta ; \phi ; \varphi ; \psi ; x\}$;

$q = 5$ (absolute coordinates)

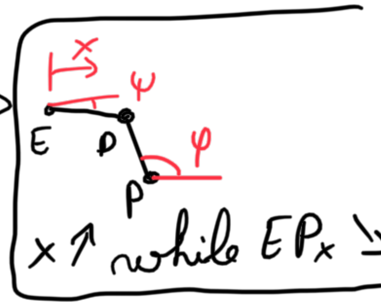
n° of constrains equations (necessary)

$r = q - N = 5 - 1 = 4$

Constrain equations

expressed with respect to the fixed joint P

to join E
$$\begin{cases} x = -ED\cos\Psi - DP\cos\varphi \\ EP_y = DP\sin\varphi + ED\sin\Psi \end{cases}$$



to joint A
$$\begin{cases} AP_x = AB\cos\theta + BC\cos\phi + CP\cos\varphi \\ AP_y = -AB\sin\theta + BC\sin\phi + CP\sin\varphi \end{cases}$$

3

Virtual work principle $d\mathcal{T} = \sum_i \bar{R}_i \, dx_i = 0$

$$d\mathcal{T} = F\,dx + C\,d\theta = 0 \iff \boxed{\frac{F}{C} = \frac{-d\theta}{dx}}$$

try to find the time evolution of $(x;\theta)$ using the coordinate partitioning method: $d\Phi = \sum_i \frac{d\Phi}{dq_i} dq_i = 0$

$$\begin{cases} ① \; dx = ED\sin\Psi\,d\Psi + DP\sin\varphi\,d\varphi \\ ② \; 0 = DP\cos\varphi\,d\varphi + ED\cos\Psi\,d\Psi \\ ③ \; 0 = -AB\sin\theta\,d\theta - BC\sin\phi\,d\phi - CP\sin\varphi\,d\varphi \\ ④ \; 0 = -AB\cos\theta\,d\theta + BC\cos\phi\,d\phi + CP\cos\varphi\,d\varphi \end{cases}$$

$$④ : d\phi = \frac{AB \cos\theta \, d\theta - CP \cos\varphi \, d\varphi}{BC \cos\phi}$$

$$③ : 0 = -AB \sin\theta \, d\theta - \cancel{BC}\ \cancel{\sin}\phi^{\,tg\,\phi} \, \frac{AB \cos\theta \, d\theta - CP \cos\varphi \, d\varphi}{\cancel{BC \cos\phi}} - CP \sin\varphi \, d\varphi$$

$$\Leftrightarrow 0 = \left(-AB \sin\theta - AB \, tg\,\phi \cos\theta\right) d\theta + \left(CP \, tg\,\phi \cos\varphi - CP \sin\varphi\right) d\varphi$$

$$\Leftrightarrow d\varphi = \frac{\left(AB \sin\theta + AB \, tg\,\phi \cos\theta\right) d\theta}{CP \, tg\,\phi \cos\varphi - CP \sin\varphi} = M \, d\theta$$

$$② : \quad d\psi = \frac{-DP \cos\varphi}{ED \cos\psi} \, d\varphi = \frac{-DP \cos\varphi}{ED \cos\psi} \, M \, d\theta$$

$$① : \; dx = \cancel{ED}\ \cancel{\sin}\psi^{\,tg\,\psi} \, \frac{-DP \cos\varphi}{\cancel{ED \cos\psi}} \, M \, d\theta + DP \sin\varphi \, M \, d\theta$$

$$dx = \left(-DP \, tg\,\psi \cos\varphi + DP \sin\varphi\right) M \, d\theta$$

$$\frac{-d\theta}{dx} = \frac{1}{\left(DP \, tg\,\psi \cos\varphi - DP \sin\varphi\right) M} = F/C$$

Newtonian dynamics & SDOF

Pinned rigid bar system

For the following mechanism:

1. If you had to choose, would you derive the equation of movement using Newtonian or Lagrangian dynamics? Why?
2. Write the equation of motion of the system assuming small angles of rotation.
3. From the equation of motion, give the expression of the damped resonant frequency of the system.

Additional details for the exercise:

- Z represents an imposed displacement
- The moment of inertia of the bar is $I = (ml^2)/3$ when computed at its edge.

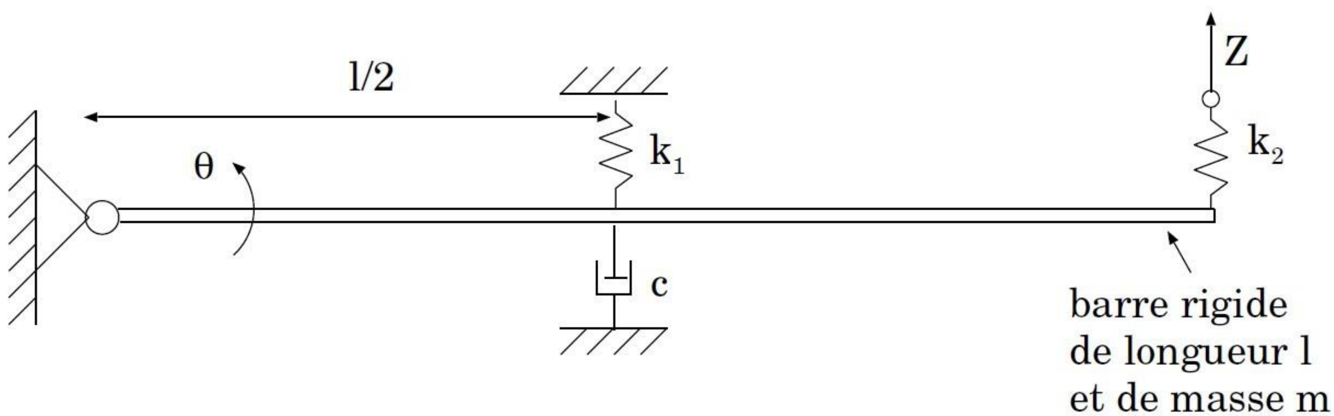


Figure 3: Rigid bar pinned

1 Newtonian dynamics applied to a system of bars is quite difficult due to the necessity of calculating the interaction forces between each bar, however in this exercice we have only one bar with external forces applied to it. Therefore using only one equilibrium formula we can derive the equation of motion, unlike Lagrange dynamics that requires multiple derivatives.

2

Force damper $= c \cdot v$ → velocity at the attached point

Force spring $= k \cdot x$ → displacement at the attached point

A : pinned point of bar

Angular momentum (rotation)

$$M_G = I\dot{\theta}, \qquad \frac{dM_G}{dt} = I\ddot{\theta}$$

$$\frac{dM_A}{dt} = \cancel{m\bar{v}_G \times \bar{v}_A} + \bar{m}_{ext_A}$$

$\bar{v}_A = 0$; $\bar{m}_{ext_A}$ → applied force momentum:

$$\bar{m}_{ext_A} = \begin{cases} -\left(k_1 \frac{l}{2}\theta\right)\frac{l}{2} \\ +k_2(Z_1 - l\theta)\, l \\ -\left(c\frac{l}{2}\dot{\theta}\right)\frac{l}{2} \end{cases}$$

(Force) (lever arm)

$$\frac{ml^2}{3}\ddot{\theta} = \frac{-k_1 l^2\theta}{4} - k_2 l^2\theta + k_2 Z_1 l - \frac{c\, l^2\dot{\theta}}{4}$$

- $\frac{ml^2}{3}\ddot{\theta}$: $\frac{dM_G}{dt}$
- $\frac{-k_1 l^2\theta}{4}$: spring k_1 force momentum
- $-k_2 l^2\theta$: spring k_2 force mom. from θ
- $+k_2 Z_1 l$: spring k_2 force mom. from Z_1
- $-\frac{c\, l^2\dot{\theta}}{4}$: damper c force mom.

2

From the equation of movement

$$\Leftrightarrow m\ddot{\theta} + \frac{3}{4}k_1\theta + 3k_2\theta + \frac{3}{4}c\dot{\theta} = \frac{3k_2 Z_1}{l}$$

$$\Leftrightarrow \underbrace{m}\ddot{\theta} + \underbrace{\frac{3}{4}c}\dot{\theta} + \underbrace{\left(\frac{3}{4}k_1 + 3k_2\right)}\theta = \frac{3k_2 Z_1}{l}$$

General equation of motion of damped sys. $M\ddot{\theta} + B\dot{\theta} + K\theta = f$

Resonnant frequency (damped) $\omega_d = \omega_n\sqrt{1-\xi}$ where $\begin{cases} \xi = \dfrac{B}{2\sqrt{KM}} \\ \omega_n = \sqrt{\dfrac{K}{M}} \end{cases}$

$$\omega_n = \sqrt{\frac{\frac{3}{4}k_1 + 3k_2}{m}}$$

$$\xi = \frac{\frac{3}{4}c}{2\sqrt{\left(\frac{3}{4}k_1 + 3k_2\right)m}}$$

$$\omega_d = \sqrt{\frac{\frac{3}{4}k_1 + 3k_2}{m}\left(1 - \frac{c}{2\sqrt{\left(\frac{3}{4}k_1 + 3k_2\right)m}}\right)}$$