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# Session 2 : MDOF systems

## Exercise 1: 3DOFs system

Consider the following three degrees-of-freedom model:

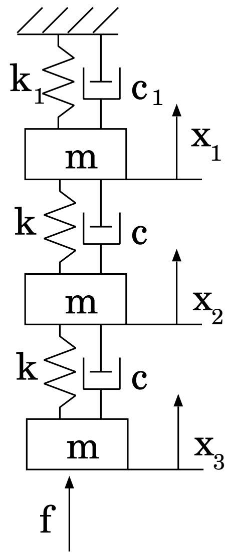



Figure 1: 3-DOFs system

a) Write the equations of motion in the time domain in a matrix form.

b) For the numerical values $m = 1\,kg$, $k = k_1 = 16\,N/m$, $c = c_1 = 0.1\,Ns/m$, compute the eigenfrequencies and the mode shapes of the conservative system. Represent the mode shapes using for example a bar graph ($barh$), and check the orthogonality conditions. .

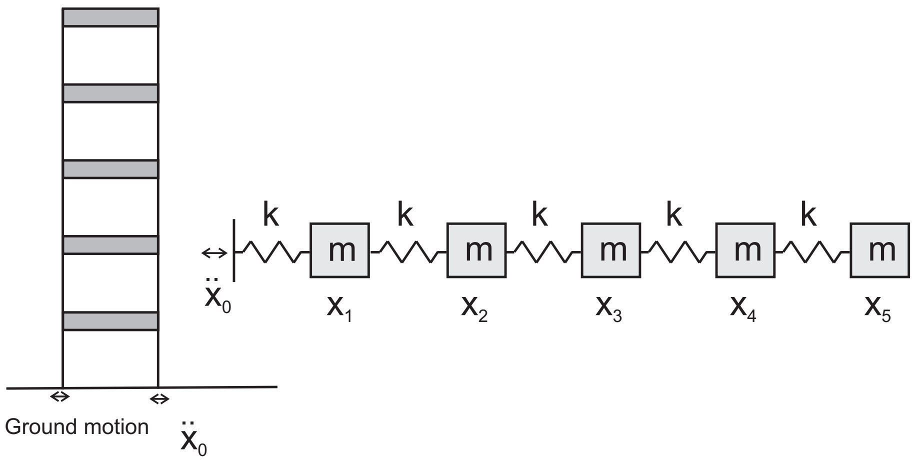
c) For the value of $k_1 = k$, compute and represent the impulse response for $x_3$ by projecting the equations of motion in the modal basis.

d) Compute the response in the frequency domain using the projection in the modal basis. Represent the Bode diagram (amplitude and phase) for the same coordinate $x_3/f$, and for the acceleration $\ddot{x_3}/f$. Is the modal damping hypothesis valid ? Multiply by a factor 5 the damping coefficient of mode 2 and plot the Bode diagram for $x_3/f$ on the same graph as with the initial value of the damping. Comment.

e) Consider the case when $c_1$=0. Is the modal damping hypothesis still verified ? Draw the Bode diagram for $x_3/f$ using the full system of equations (solve frequency by frequency, without projection in the modal basis). Compare with the modal approach in which the coupling is neglected. Comment

Note: Use the `eig` function to compute the eigenfrequencies and mode shapes.

# Exercise 2 : 5 storeys building




Consider a building with five storeys, which can be approximated by a 5 dofs model as shown in the figure.

a) Write the equations of motion in the matrix form. The excitation of the system is in the form of a ground acceleration $\ddot{x_0}$.

b) Compute the mode shapes and eigenfrequencies of the building for $k = 10^9 N/m$ and $m = 70000kg$, and a fixed base. Represent the mode shapes.

c) Compute the transfer function between the relative displacement of the top floor and the ground acceleration using the decomposition in the modal domain and explicit analytical expressions. Take a value of $\xi_i = 0.01$ for all five modes

d) Consider the first earthquake of session 1. Compute and represent the time domain response (relative displacement) of the top floor of the building, excited by the earthquake. Represent also the absolute acceleration.