VIBRATION TESTING PART II

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DATA ACQUISITION

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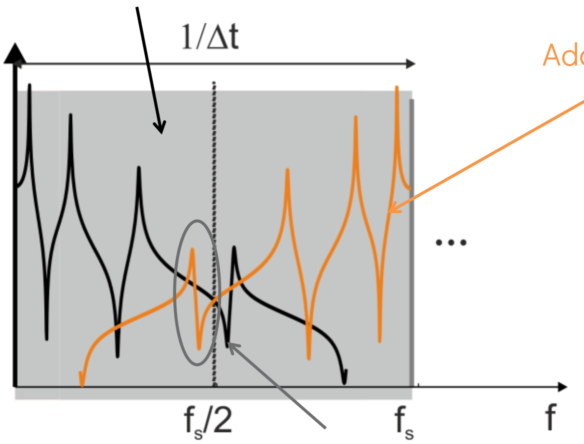
Sampling and aliasing



Part of the additional signal 'falls back' on the frequency band of interest.
The frequency of the fourth resonance is wrong due to aliasing 3

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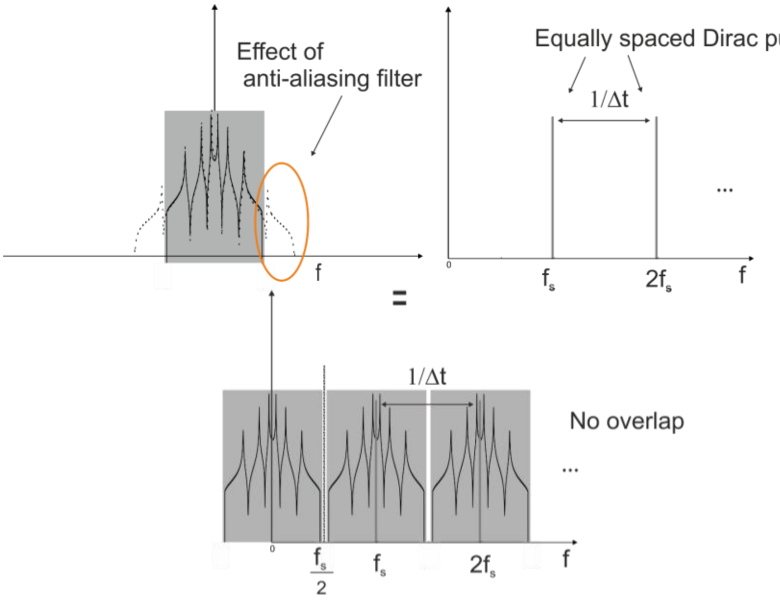
Anti-aliasing filters



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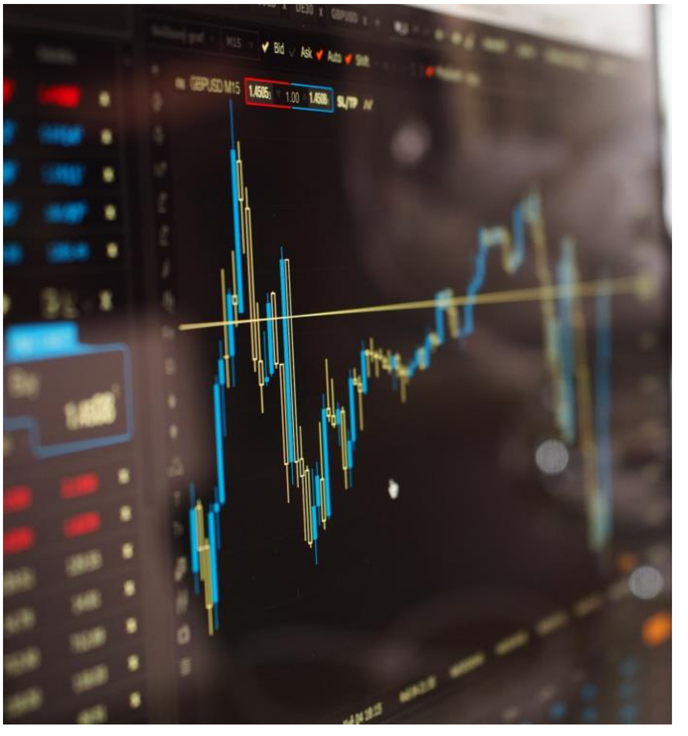
ESTIMATING FRFs



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Measuring FRFs

f(t)
(hammer, shaker + force sensor)

x(t)
(accelerometer, laser, ...)

$$H(\omega) = \frac{X(\omega)}{F(\omega)}$$

?

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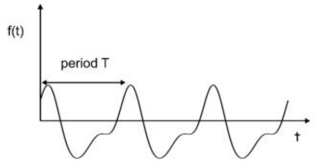
Types of excitation signals

Shaker

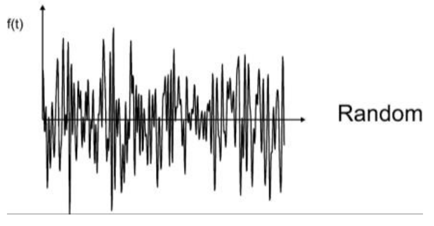


Periodic

Shaker,
impact hammer

Transient

Shaker



Random

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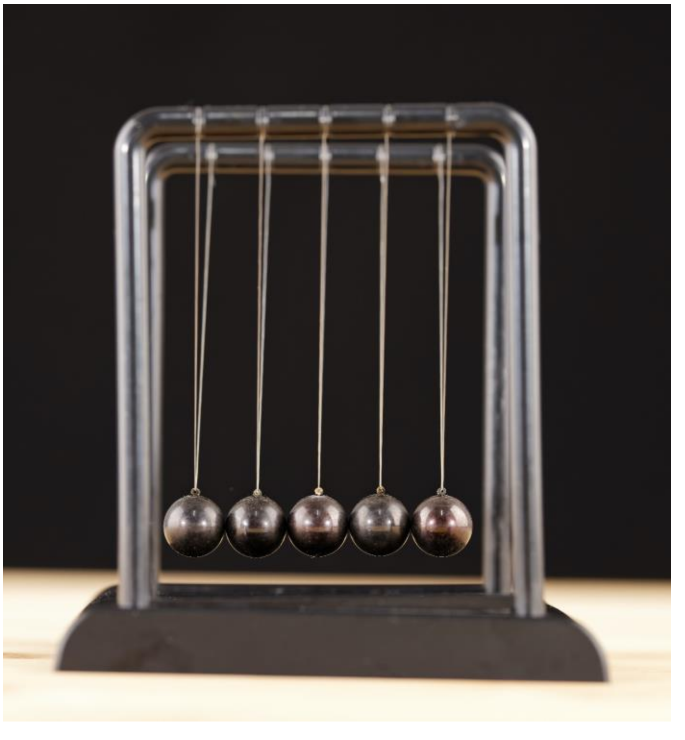
PERIODIC SIGNALS



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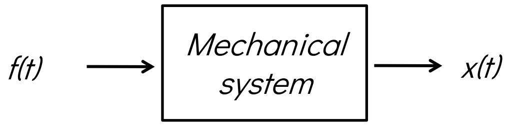
Periodic excitation



Both signals have a period T ⟶ Discrete Fourier Transform

$$x(t) = \sum_{n=-\infty}^{n=\infty} X_n e^{in\omega_0 t} \qquad X_n = \frac{1}{T}\int_0^T x(t)e^{-in\omega_0 t}dt$$

$$f(t) = \sum_{n=-\infty}^{n=\infty} F_n e^{in\omega_0 t} \qquad F_n = \frac{1}{T}\int_0^T f(t)e^{-in\omega_0 t}dt$$

⟶ $H_n = \frac{X_n}{F_n}$ Defined at discrete frequencies multiples of ω_0

In practice, signals are sampled : integral ⟶ sum
frequency interval : [0 f_s/2]

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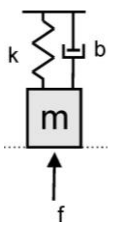
Periodic signals and averaging : examples

k=1 N/m
m= 1kg
b = 0.05 Ns/m

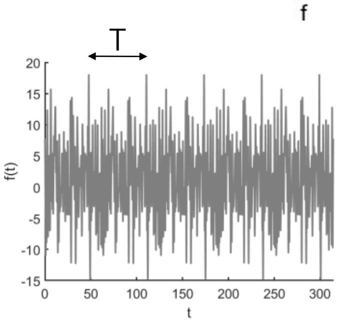


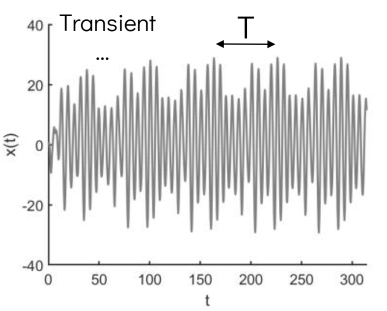
Periodic input signal

$$f(t) = \sum_{i=0}^{n} cos(\omega_0 t + \phi)$$

$$\omega_0 = 0.1 rad/s, T = 62.83 s$$

n=64, random phases





Output signal is periodic after transient

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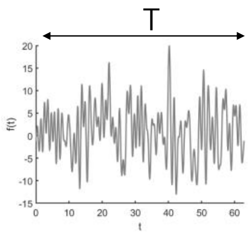
Periodic signals and averaging: examples

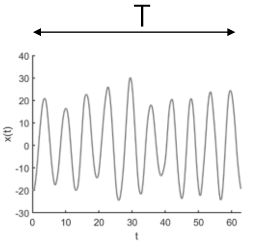


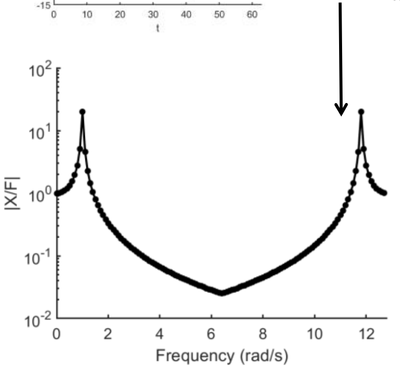
Signals sampled with Δt=T/128
(n=128 samples)

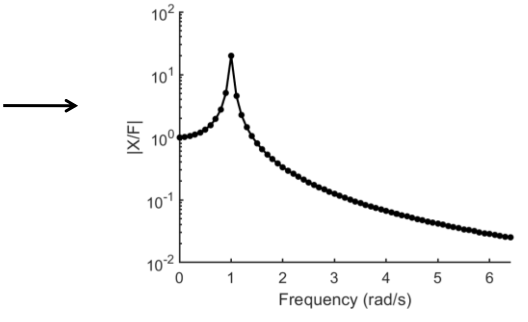
Discrete Fourier Transform

$$\Delta\omega = \frac{2\pi}{T} = 0.1 rad/s$$

$$\omega_{max} = 128 * 0.1 rad/s = 12.8 rad/s$$







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Averaging

In reality, signals are noisy. Better estimates are obtained using averaging.
In order to do that, signals are measured over several periods.

FRF measured over period *i*

$$H_{ni} = \frac{X_{ni}}{F_{ni}} \qquad i = 1 \dots m$$

Average FRF

$$H_n = \frac{\frac{1}{m}\sum_{i=1}^{m} X_{ni}}{\frac{1}{m}\sum_{i=1}^{m} F_{ni}}$$

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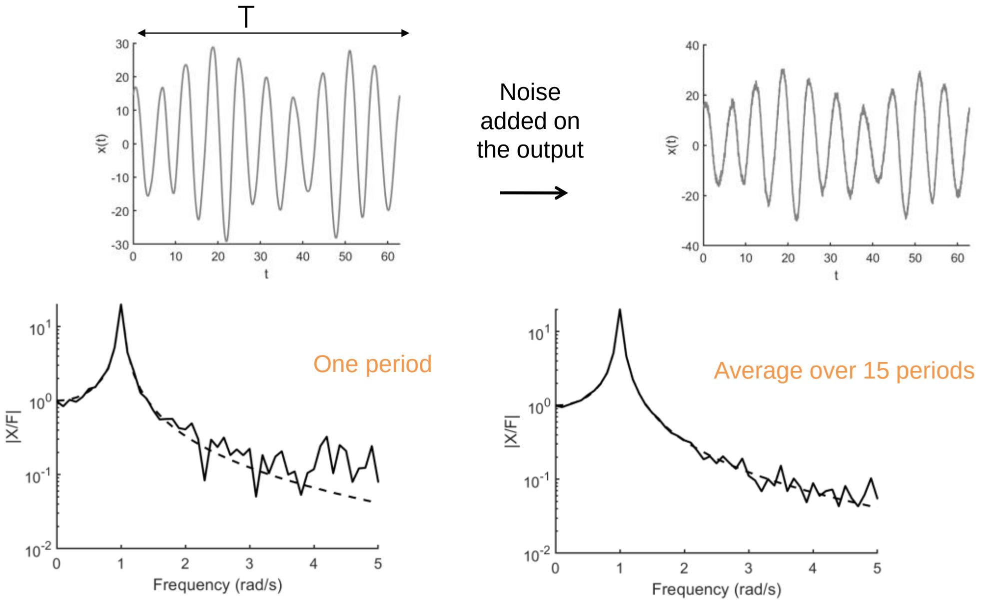
Periodic signals and averaging : examples



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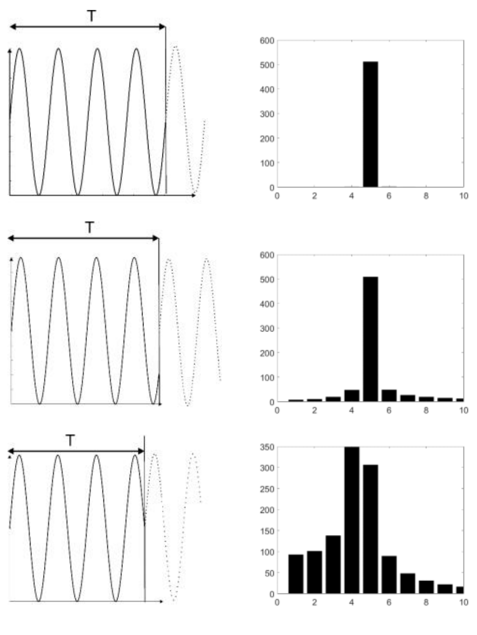
Effect of bad synchronization : leakage



No leakage

Small leakage

Large leakage

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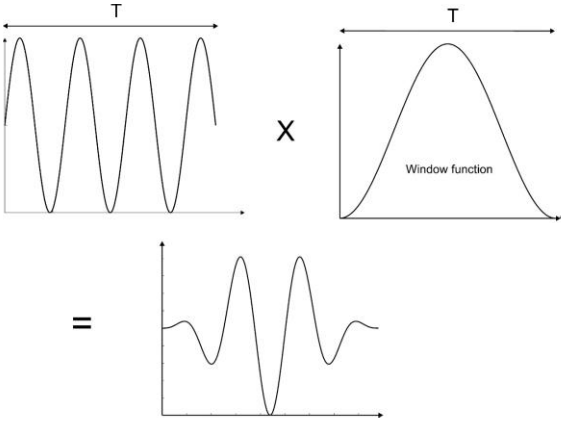
Decreasing leakage

The signal is multiplied by a **windowing function** also called **window** before the Fourier Transform



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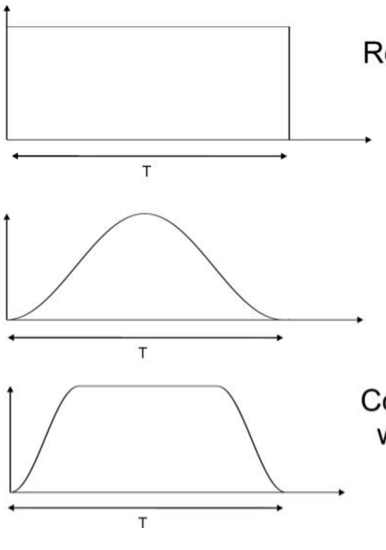
Window types



Rectangular (boxcar) window

Hanning window

Cosine-taper (Tukey) window

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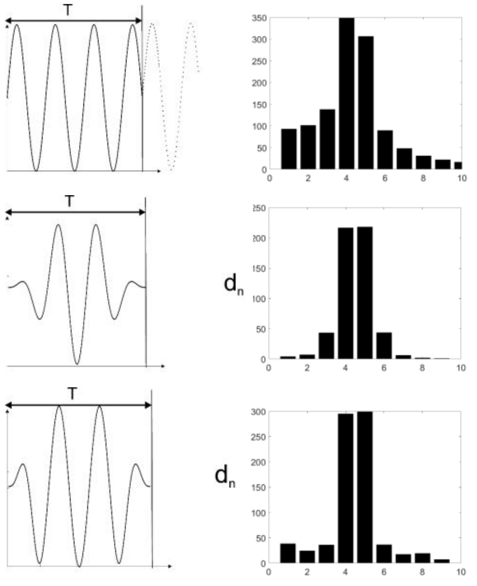
Effect on leakage



No window (rectangle)

Hanning

Tukey (cosine tapper)

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TRANSIENT SIGNALS

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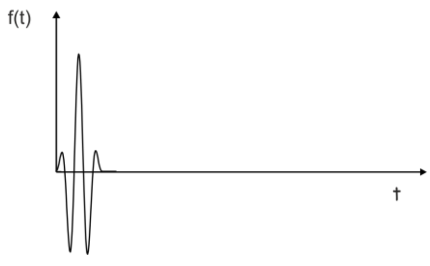
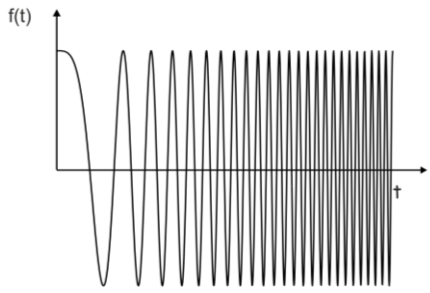
Transient signals

Impulse



Sine burst



Chirp

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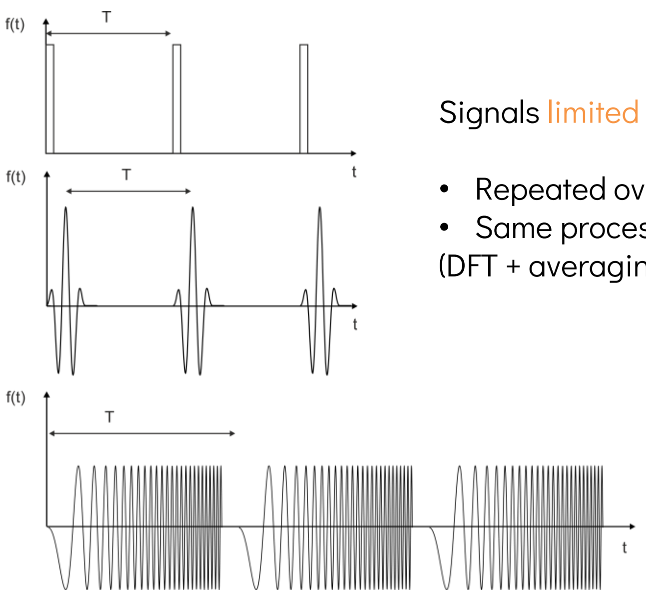
Transient signals



Signals limited in time

- Repeated over several periods
- Same processing as periodic signals

(DFT + averaging)

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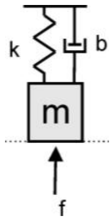
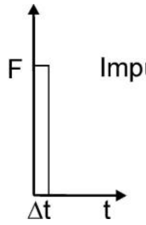
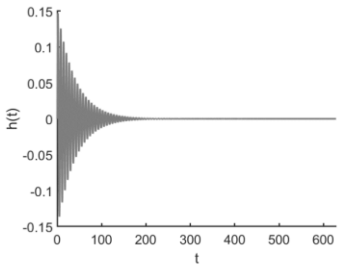
Transient signals : impulse response

k=1 N/m
m= 1kg
b = 0.05 Ns/m

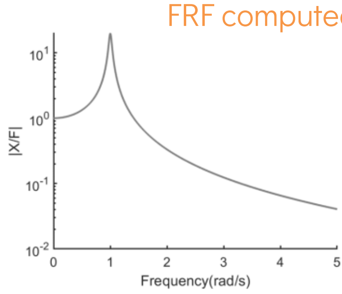




Impulse=FΔt



FRF computed using DFT



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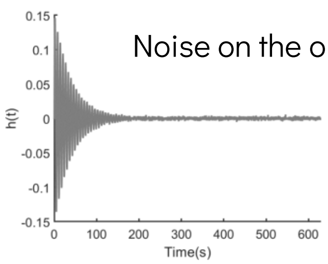
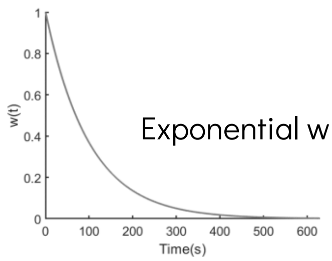
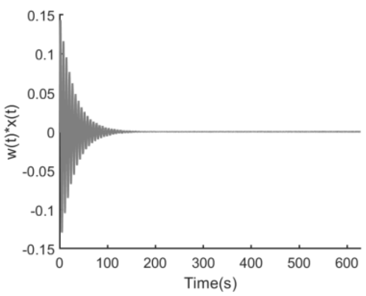
Transient signals : impulse response and noise



Noise on the output



Exponential window



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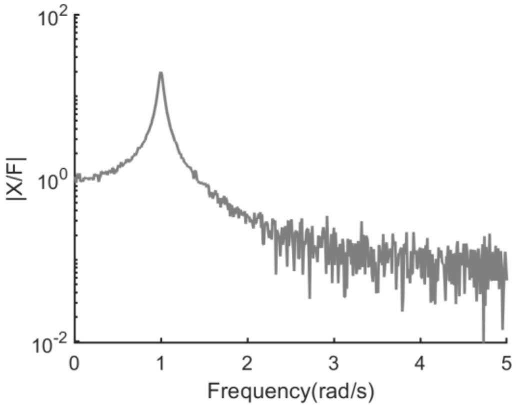
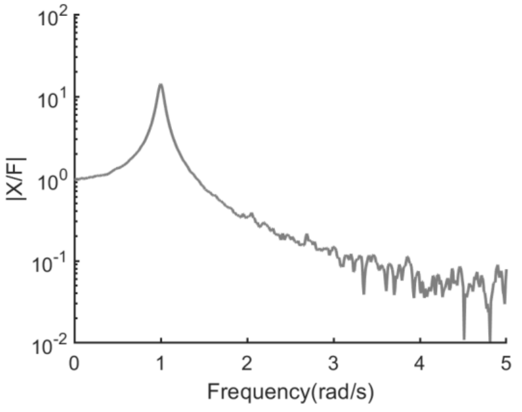
Transient signals : impulse response and noise

No window



Exponential window



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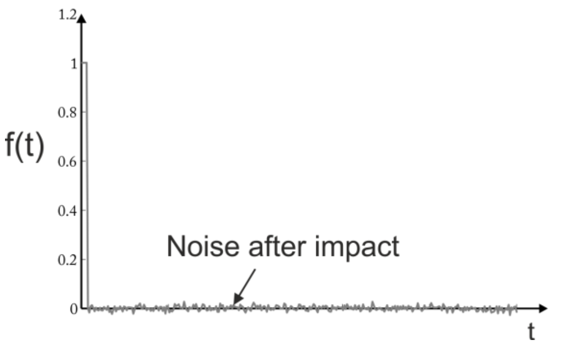
Transient signals : impulse response and noise

Force sensor measurement



Force window

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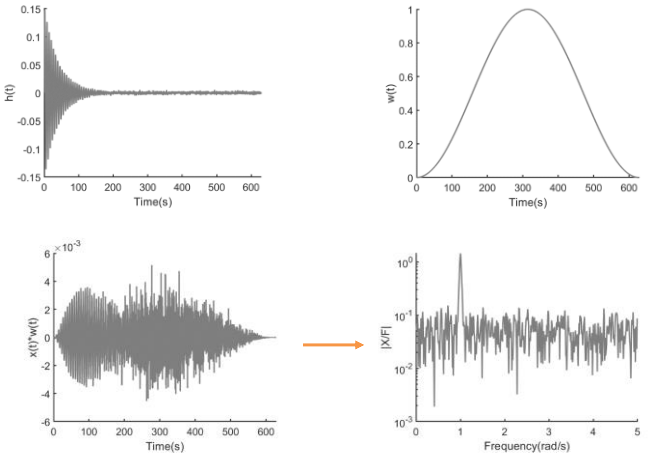
Transient signals : impulse response and noise

Hanning window



FRF computed using DFT

Hanning should never be used with impulse excitations !

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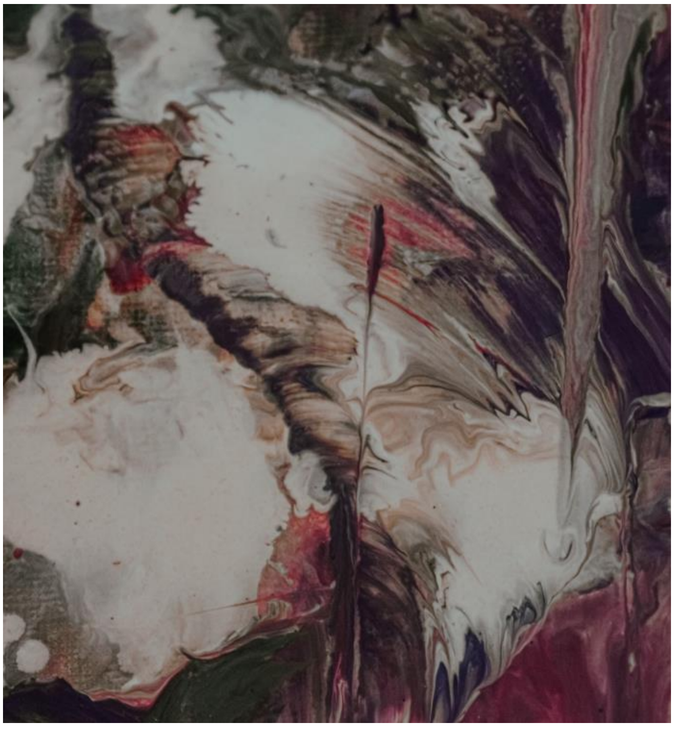
RANDOM SIGNALS



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Random signals : definitions

The Autocorrelation and Cross-correlation functions

$$R_{ff}(\tau) = E[f(t)\,f(t+\tau)]$$
$$R_{fx}(\tau) = E[x(t)\,f(t+\tau)]$$
$$R_{xf}(\tau) = E[f(t)\,x(t+\tau)]$$
$$R_{xx}(\tau) = E[x(t)\,x(t+\tau)]$$

Expected (average) value

$$E[x(t)] = \int_{-\infty}^{\infty} x(t)dt$$

The Power Spectral Density and Cross Spectral Density

$$S_{ff}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{ff}(\tau)e^{-i\omega\tau}d\tau \qquad S_{xf}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{xf}(\tau)e^{-i\omega\tau}d\tau$$

$$S_{fx}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{fx}(\tau)e^{-i\omega\tau}d\tau \qquad S_{xx}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{xx}(\tau)e^{-i\omega\tau}d\tau$$

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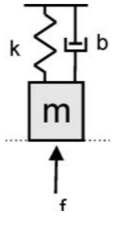
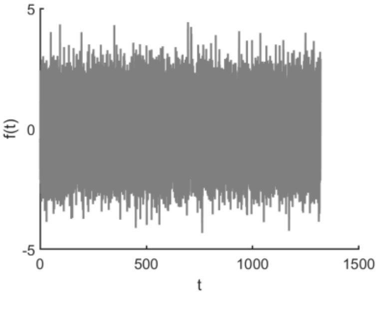
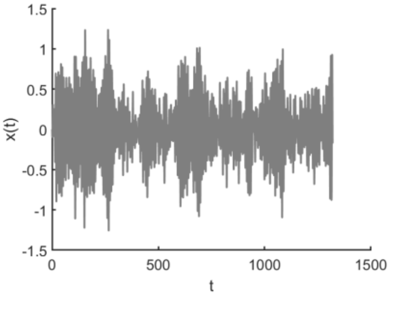
Random signals : definitions

k=1 N/m
m= 1kg
b = 0.05 Ns/m







Output is random (no periodicity …)

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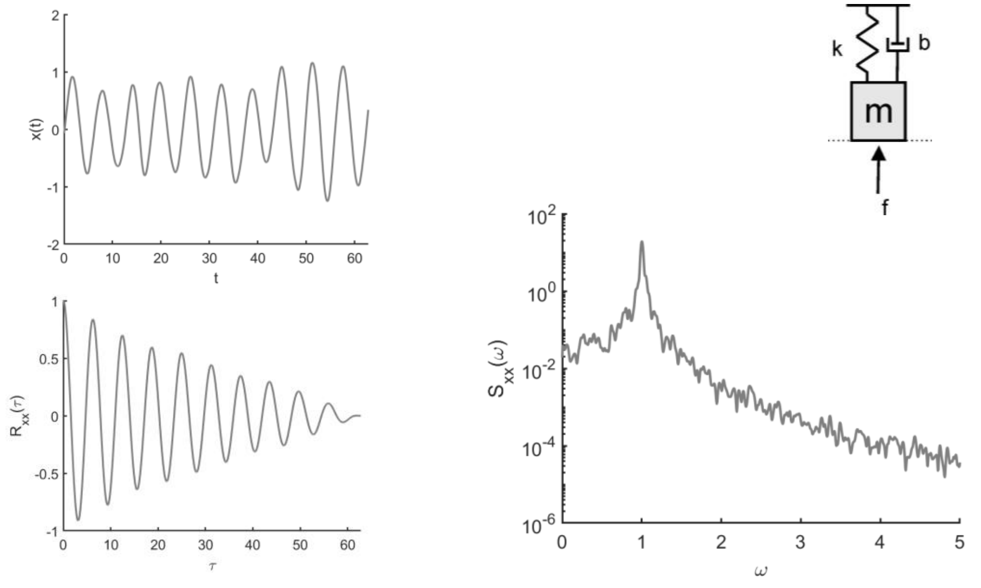
Random signals : definitions



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Random signals : FRF estimation

$$S_{ff}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{ff}(\tau)e^{-i\omega\tau}d\tau \qquad S_{xf}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{xf}(\tau)e^{-i\omega\tau}d\tau$$

$$S_{fx}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{fx}(\tau)e^{-i\omega\tau}d\tau \qquad S_{xx}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{xx}(\tau)e^{-i\omega\tau}d\tau$$

Duhamel's integral

$$x(t) = \int_{-\infty}^{\infty} f(\tau)h(t-\tau)d\tau = f(t) * h(t)$$

$$\begin{aligned} S_{xx}(\omega) &= H^*(\omega)S_{fx}(\omega) \\ S_{fx}(\omega) &= H(\omega)S_{ff}(\omega) \\ S_{fx}(\omega) &= S^*_{xf}(\omega) \\ S_{xx}(\omega) &= |H(\omega)|^2 S_{ff}(\omega) \end{aligned}$$

$$H(\omega) = \frac{S_{fx}(\omega)}{S_{ff}(\omega)} = H_1(\omega)$$

$$H(\omega) = \frac{S_{xx}(\omega)}{S_{xf}(\omega)} = H_2(\omega)$$

Coherence function

$$\gamma^2(\omega) = \frac{H_1(\omega)}{H_2(\omega)} = \frac{|S_{fx}(\omega)|^2}{S_{ff}(\omega)S_{xx}(\omega)}$$

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Random signals : FRF estimation

FRF estimates

$$H(\omega) = \frac{S_{fx}(\omega)}{S_{ff}(\omega)} = H_1(\omega)$$

$$H(\omega) = \frac{S_{xx}(\omega)}{S_{xf}(\omega)} = H_2(\omega)$$

Coherence

$$\gamma^2(\omega) = \frac{H_1(\omega)}{H_2(\omega)} = \frac{|S_{fx}(\omega)|^2}{S_{ff}(\omega)S_{xx}(\omega)}$$

- Coherence=1 if measurement is perfect.
- Coherence usually drops :
    - around eigenfrequencies
    - when measurements are very noisy
    - when the structure is non-linear

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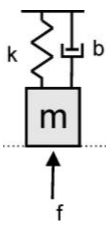
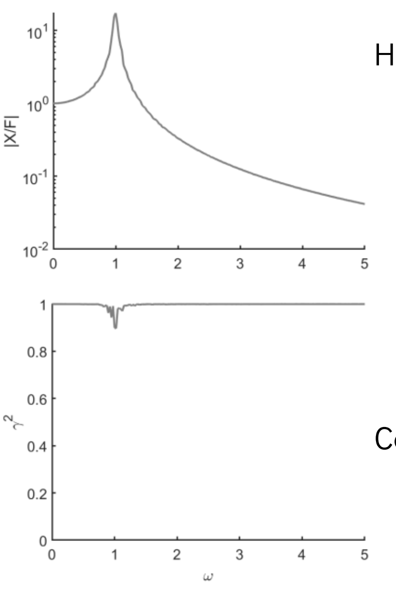
Random signals : PSD estimation





H_1 Estimator

Coherence

Hanning window used to reduce leakage
(signal is not periodic !)

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Random signals : PSD estimation

$$S_{xx}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{xx}(\tau)e^{-i\omega\tau}d\tau$$

$$R_{xx}(\tau) = E[x(t)\,x(t+\tau)]$$

$$E[x(t)\,x(t+\tau)] = \int_{-\infty}^{\infty} x(t)\,x(t+\tau)dt = x(t) * x(-t)$$

→ $$S_{xx}(\omega) = X(\omega)X^*(\omega)$$

In the same way :

$$S_{ff}(\omega) = F(\omega)F^*(\omega)$$

$$S_{xf}(\omega) = X(\omega)F^*(\omega)$$

These FRF estimates can be computed for <u>any type of signal</u>, including random signals, impulse excitation, …