DAMPING

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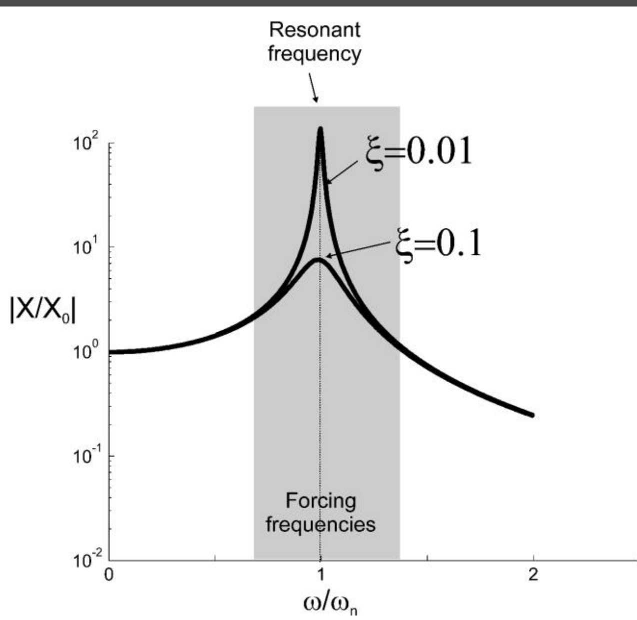
Adding damping



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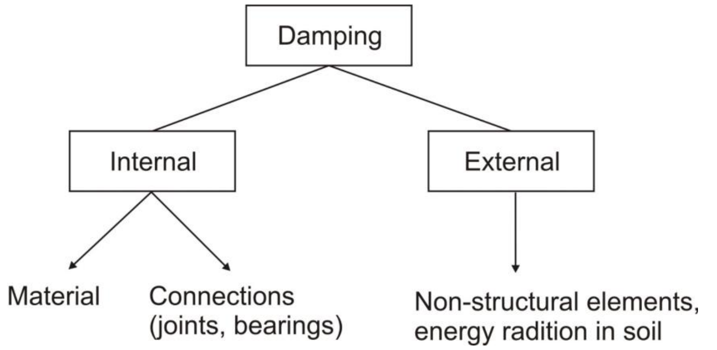
Types and origin of damping

Damping = dissipation of energy



→ Local damping models

[Vibration problems in structures, H. Bachman, 1995]

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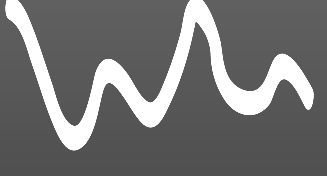
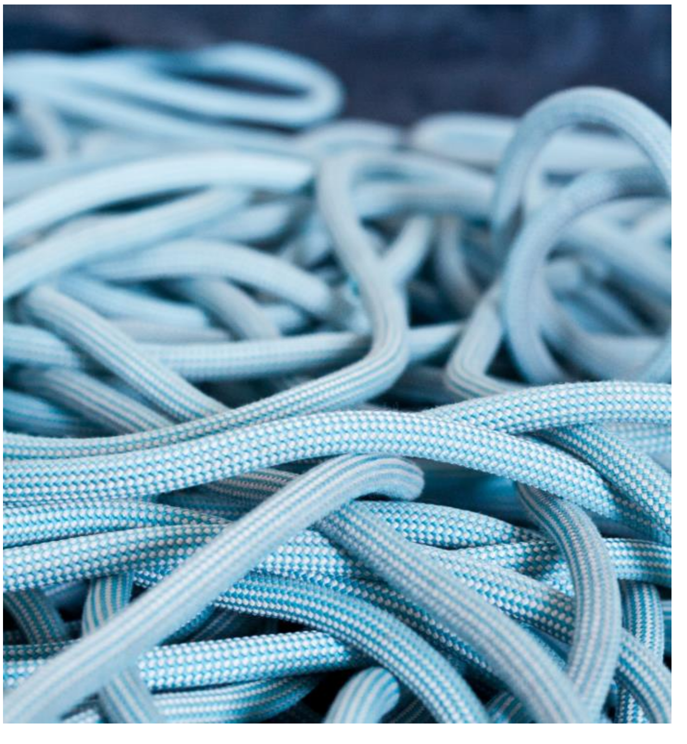
ADDING MATERIAL DAMPING





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Material damping

Viscous damping

$C_i = \alpha_i K_i$ In each material

Loss factor – Hysteretic damping

$E(1 + i\eta(\omega))$ Loss factor can be different for each material and frequency dependent

$\Rightarrow \xi_i = \frac{\eta}{2}$ For a single material

$\Rightarrow \xi_i = f(\alpha_i, \eta_i, \ldots)$ For different materials

Material	ξ
Reinforced concrete	0.004-0.012
Composite	0.002-0.003
Steel	0.001-0.002

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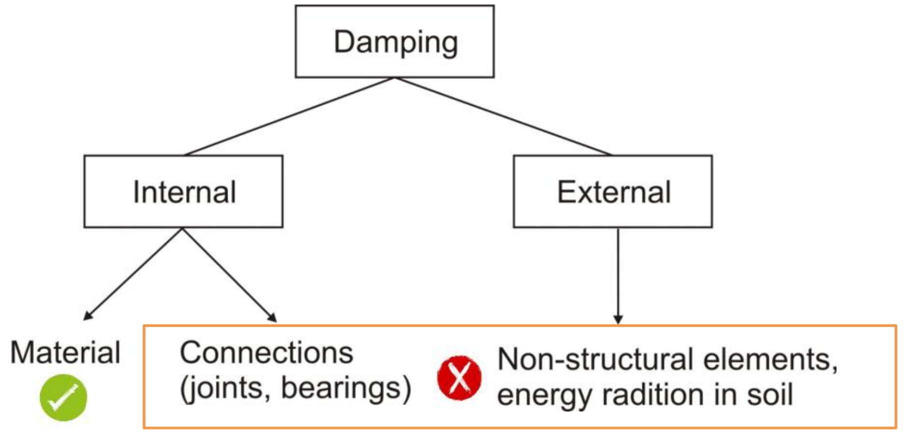
Contributions to damping



$\xi = 0.01$ is a typical value in engineering structures

Damping coefficients are usually derived from practice or measured

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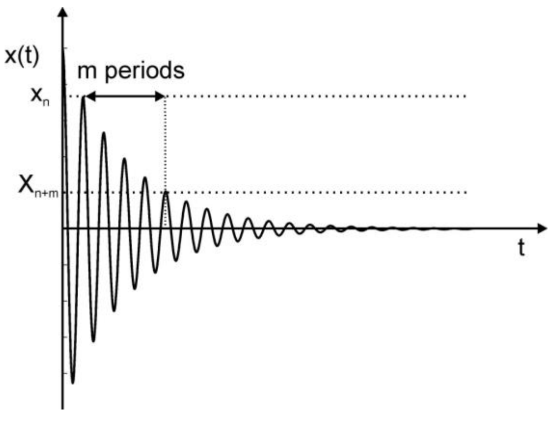
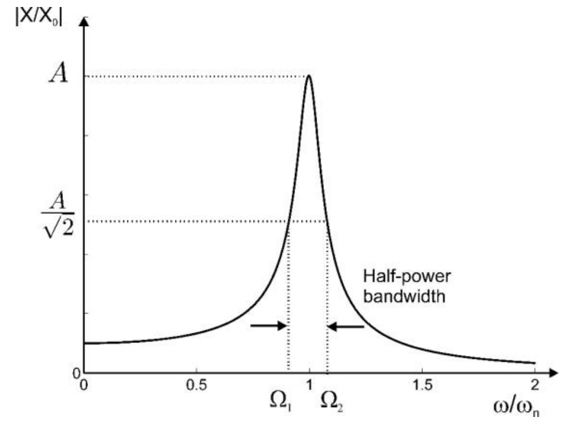
Estimation of damping

Logarithmic decrement method



Estimation of ξ in the time domain

Half-power bandwidth



Estimation of ξ in the frequency domain

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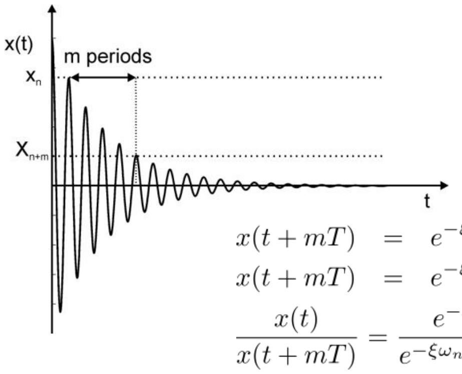
Logarithmic decrement method



Free response

$$x(t) = e^{-\xi\omega_n t}\left(Acos(\omega_d t) + Bsin(\omega_d t)\right)$$

$$x(t+mT) = e^{-\xi\omega_n (t+mT)}\left(Acos(\omega_d (t+mT)) + Bsin(\omega_d (t+mT))\right)$$

$$x(t+mT) = e^{-\xi\omega_n (t+mT)}\left(Acos(\omega_d t) + Bsin(\omega_d t)\right)$$

$$\frac{x(t)}{x(t+mT)} = \frac{e^{-\xi\omega_n t}}{e^{-\xi\omega_n (t+mT)}} = e^{\xi\omega_n (mT)}$$

$$\Lambda = ln\left(\frac{x(t)}{x(t+mT)}\right) = \xi\omega_n(mT) = \xi m\frac{2\pi}{\omega_d}\omega_n = 2m\pi\xi\frac{1}{\sqrt{1-\xi^2}}$$

$$\xi^2 << 1$$

$$\xi = \frac{1}{2\pi m}\Lambda$$

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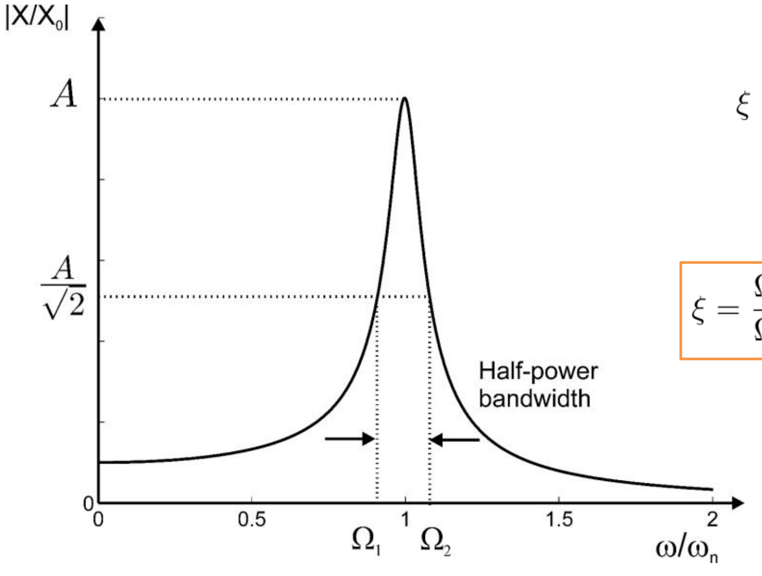
Half-power bandwidth



$$\xi < 0.1$$

$$\xi = \frac{\Omega_2 - \Omega_1}{\Omega_2 + \Omega_1}$$

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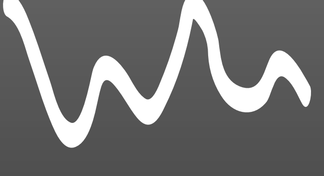
DAMPING TREATMENTS



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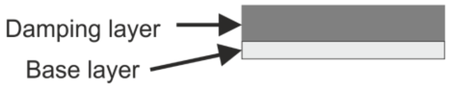
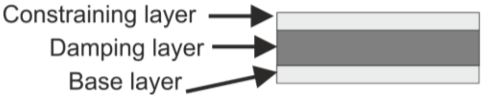
Types of damping treatments

Unconstrained (free) layer damping



Extensional damping

Constrained layer damping



Extensional and shear damping

→ Broad band effect

Unconstrained damping layer

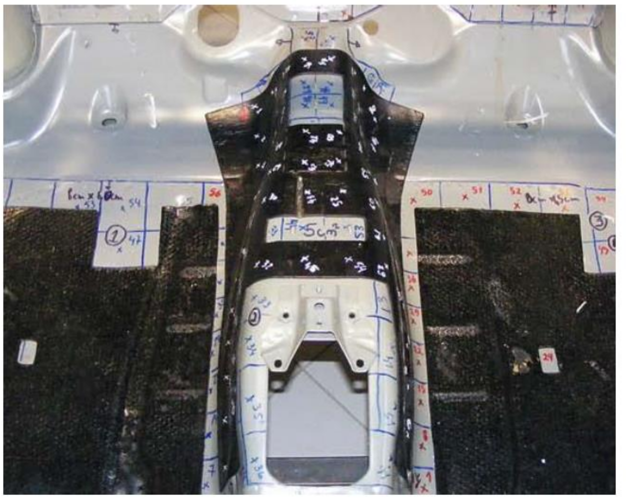


Fig. 1 – Asphalt melt sheet applied on floorpan of an automobile.

[Polycarpo 2013]

Constrained damping layer

https://www.youtube.com/watch?v=4wxL8FRCk6I

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Damping treatments

https://www.youtube.com/watch?v=MpkUbizbBjI

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Damping treatments

https://www.youtube.com/watch?v=n56oSxd3jV0

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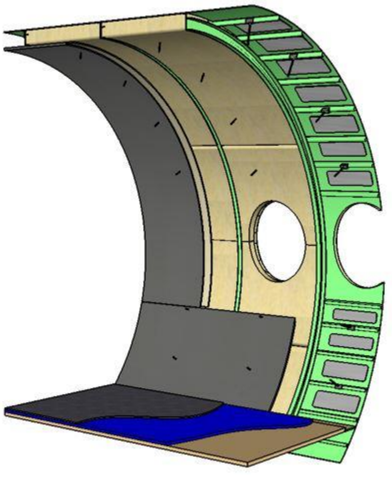
Damping treatments



www.earglobal.com

Soundproofingcompany.com

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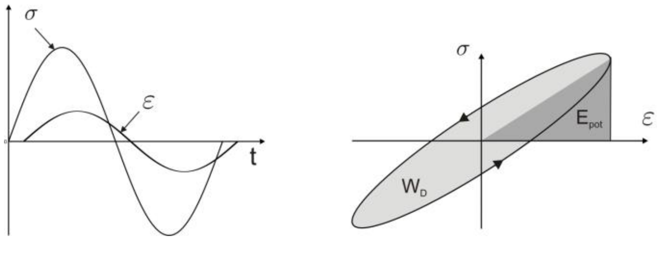
EQUIVALENT DAMPING MODELS

Hystheresis loop

When dissipation is present, the stress is not in phase with the strain, which results in a hysteresis loop



[Vibration Problems in Structures, CEB, 1991]

The mechanical energy dissipated in one cycle per unit volume is given by the area inside the loop

$$W_D = \int_0^T \sigma\dot{\varepsilon}dt \quad = \int \sigma d\varepsilon$$ T=period

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Damping factor

The damping factor of a material is proportional to the ratio of energy dissipated in one cycle to the maximum strain potential energy

$$\Psi = \frac{1}{2\pi}\frac{W_D}{E_{pot}}$$

The damping factor of the structure is given by (V is the volume of the structure) :

$$\Psi_S = \frac{\int_V \Psi dV}{V} = \frac{1}{2\pi}\frac{W_{DS}}{E_{potS}}$$

For a homogeneous structure, we have

$$\Psi = \Psi_S$$

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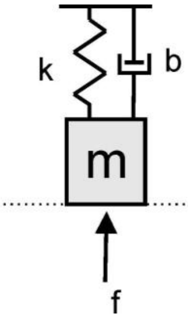
Damping factor of a SDOF with viscous damping



$$m\ddot{x} + b\dot{x} + kx = f$$

$$W_D = \int_0^T b\dot{x}\dot{x}dt$$

$$x(t) = |X|\cos(\omega t)$$

$$W_D = \int_0^T \omega^2 b|X|^2 \sin^2(\omega t)dt$$

$$= \omega^2 b|X|^2 \frac{T}{2}$$

$$T = \frac{2\pi}{\omega}$$

$$= \pi b\omega|X|^2$$

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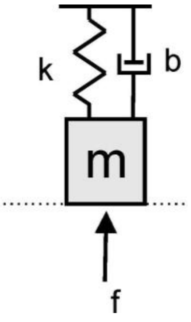
Damping factor of a SDOF with viscous damping



$$E_{pot} = \frac{1}{2} k x_{max}^2$$

$$x(t) = |X| \cos(\omega t)$$

$$E_{pot} = \frac{1}{2} k |X|^2$$

$$\Psi_S = \frac{1}{2\pi} \frac{W_{DS}}{E_{potS}} = \frac{\omega b}{k}$$

At resonance : $\omega = \sqrt{\frac{k}{m}}$ → $\Psi_S = \frac{b}{\sqrt{km}} = 2\xi$

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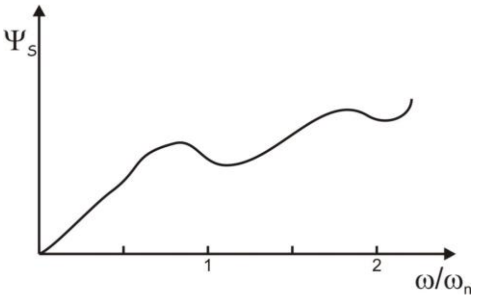
Equivalent viscous damping

Example of energy dissipation in a real structure



$$\Psi_S = \frac{1}{2\pi} \frac{W_{DS}}{E_{potS}} = \frac{\omega b}{k}$$

$$b(\omega) = \frac{k \Psi_S(\omega)}{\omega}$$

$$m\ddot{x} + b(\omega)\dot{x} + kx = f$$

Frequency dependent damping coefficient
-> difficult to use for time domain computations

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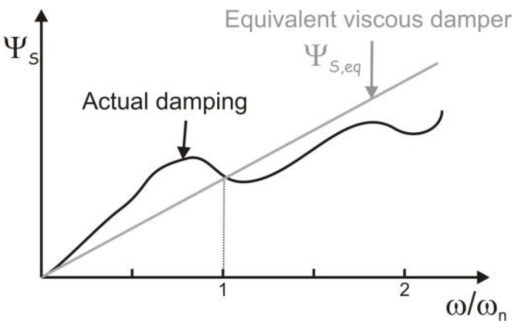
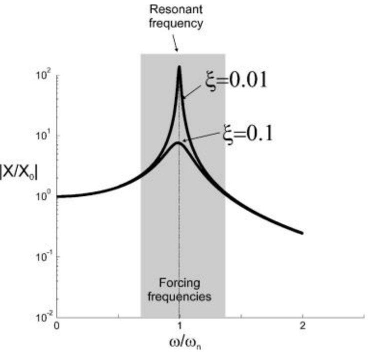
Equivalent viscous damping





$$\Psi_{S,eq} = \frac{\omega b_{eq}}{k} \longrightarrow \Psi_{S,eq} = \Psi_S \quad \text{at} \quad \omega = \omega_n$$

$$\longrightarrow \boxed{b_{eq} = \Psi_S(\omega_n)\sqrt{km}}$$

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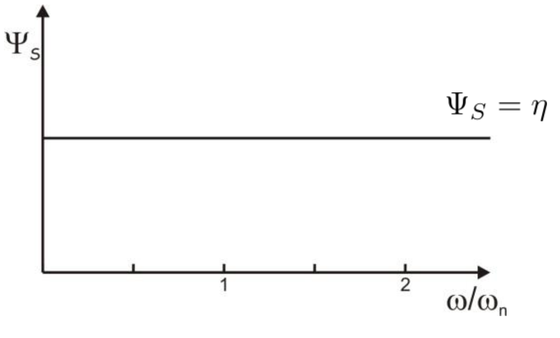
Equivalent viscous damping for hysteretic damping

Hysteretic (constant damping)



$$\boxed{b_{eq} = \eta\sqrt{km}}$$

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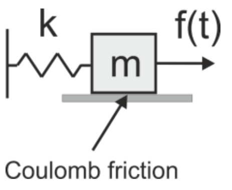
Coulomb friction damping



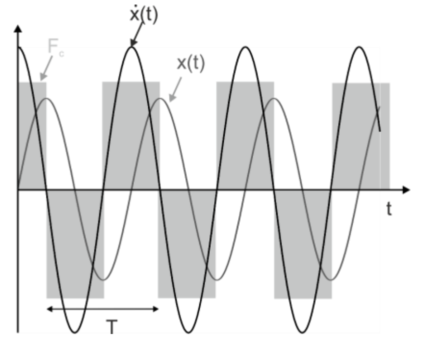
$$m\ddot{x} + F_c sgn(\dot{x}) + kx = f$$

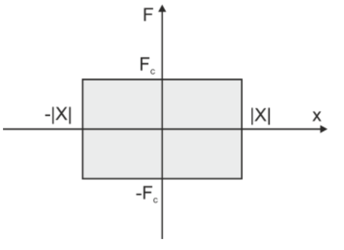
$$W_D = \int_0^T F_c \dot{x} dt = 4\int_0^{T/4} F_c \dot{x} dt = 4F_c|X|$$

$$\Psi_S = \frac{1}{2\pi}\frac{4F_c|X|}{\frac{1}{2}k|X|^2} = 4\frac{F_c}{\pi k|X|}$$

$$b_{eq} = \frac{4F_c}{\pi\omega_n|X|}$$

Amplitude dependent damping





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