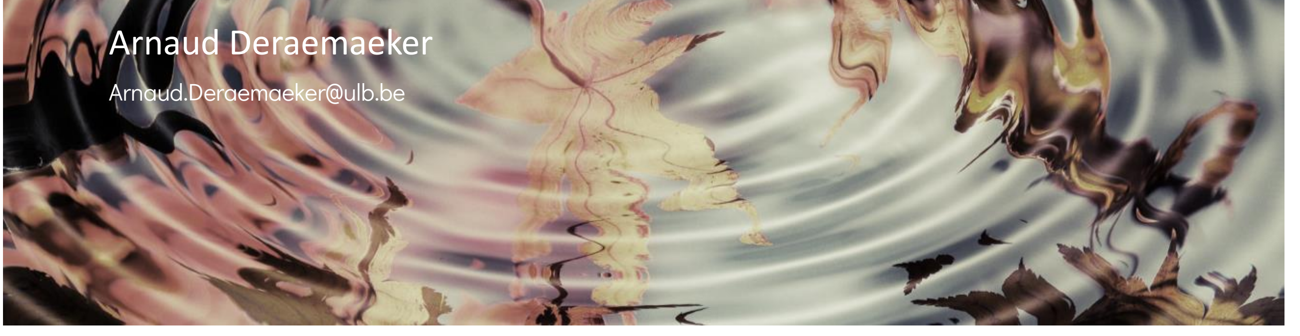
DYNAMIC RESPONSE



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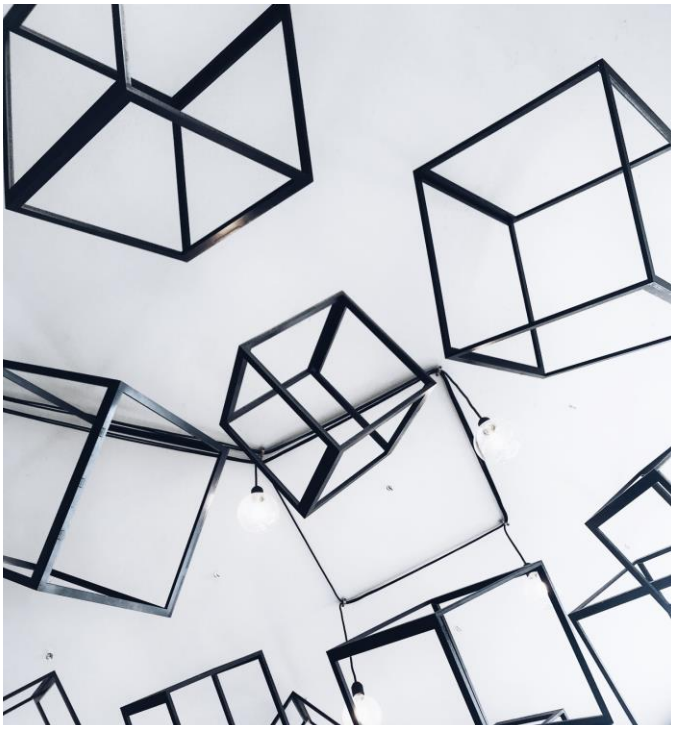
GENERAL METHODOLOGY



2

General methodology

Structure
Geometry
Material Properties
Boundary conditions
Dynamic excitations

→

Model
Mass-Spring
Continuous
Finite Element

→

Modal properties
Natural frequencies
Mode Shapes

Modal basis projection
Truncation

→

Response computation
SDOF solution for each mode (modal coordinates)
Recombine (physical coordinates)

3

3

General methodology

Frequency domain computations

$$X(\omega) = \sum_{j=1}^{n} \frac{\psi_j^T F \psi_j}{\mu_j(\omega_j^2 - \omega^2 + 2i\xi_j\omega\omega_j)}$$

Time domain computations

$$\mu_i \ddot{z}_i + 2\mu_i\xi_i\omega_i\dot{z}_i + \mu_i\omega_i^2 z_i = F_i$$

$$h_i(t) = \frac{e^{-\xi_i\omega_i t}}{\mu_i\omega_{di}}\sin(\omega_{di}t)$$

$$z_i(t) = h_i(t) * F_i(t) \qquad x(t) = \sum_{i=1}^{n} z_i(t)\psi_i$$

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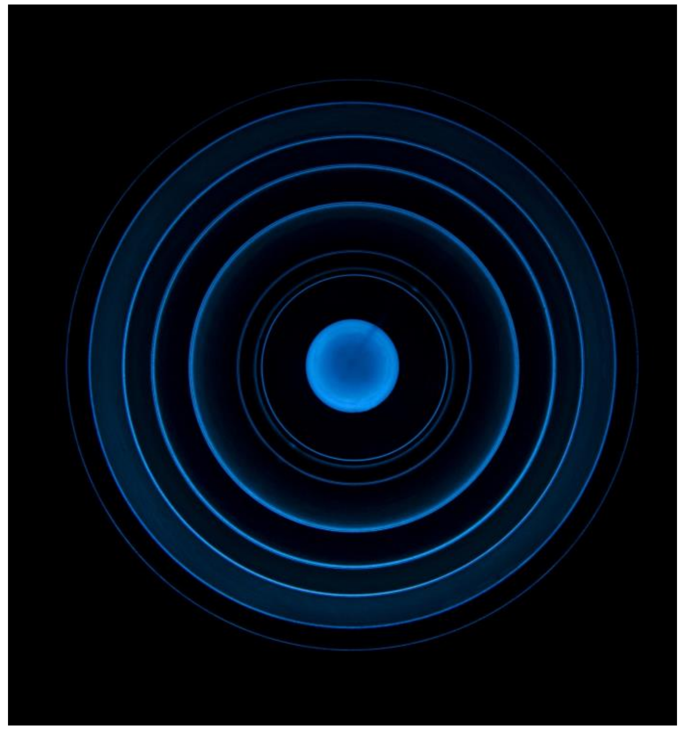
MODAL PROPERTIES



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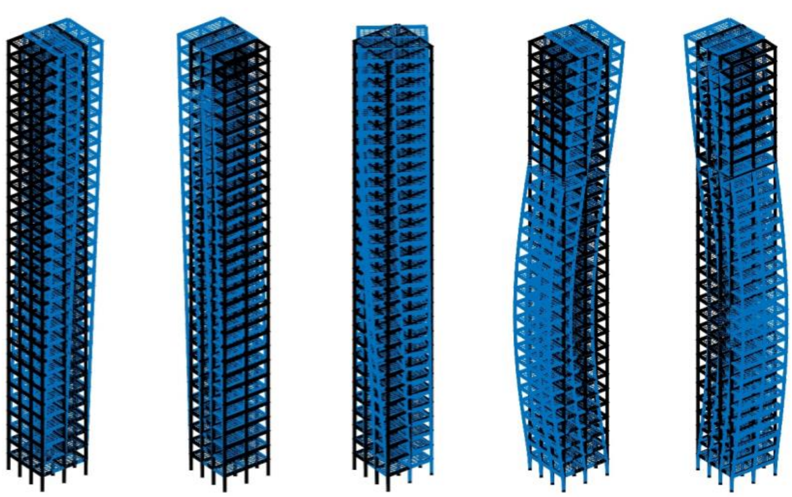
30 storey building

Mode 1 (0.24 Hz)
Mode 2 (0.27 Hz)
Mode 3 (0.40 Hz)
Mode 4 (0.79 Hz)
Mode 5 (0.89 Hz)



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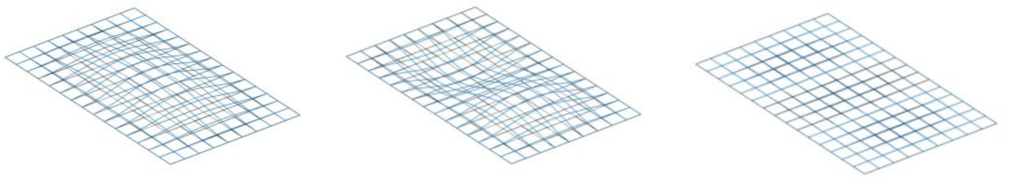
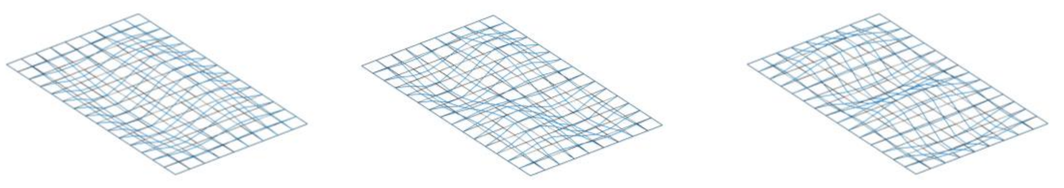
Cantilever plate mode shapes

Mode 1 (8.2 Hz) Mode 2 (11.81 Hz) Mode 3 (18.49 Hz)



Mode 4 (22.11 Hz) Mode 5 (25.38 Hz) Mode 6 (28.68 Hz)



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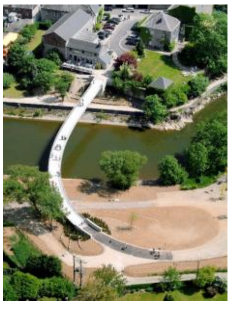
Footbridge in Durbuy

From [Bureau Greisch, 2020]



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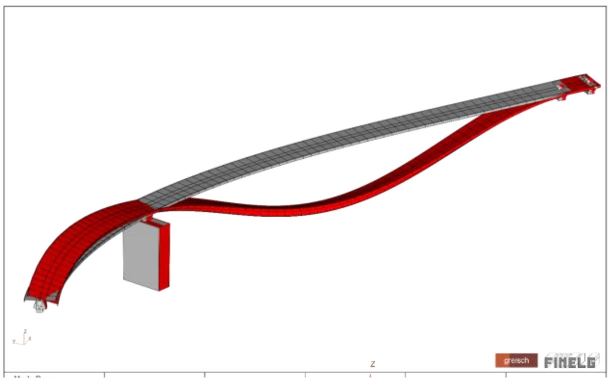
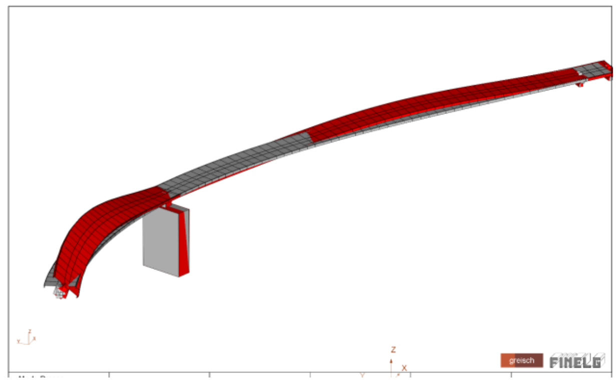
Footbridge in Durbuy

Mode 1 (2 Hz)



Mode 2 (4.16 Hz)



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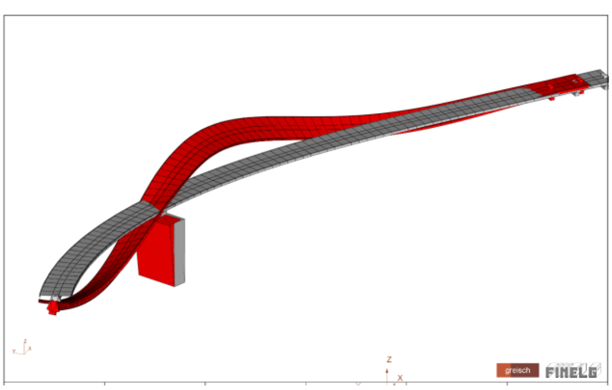
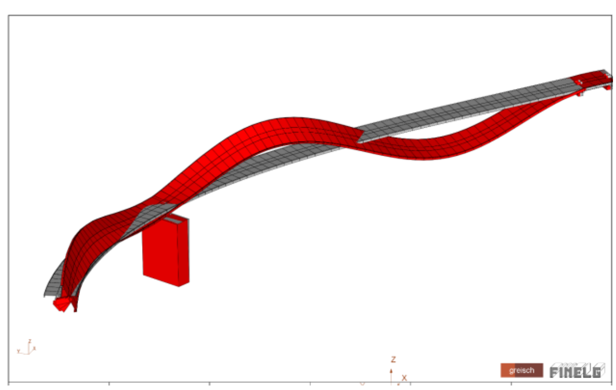
Footbridge in Durbuy

Mode 3 (4.41 Hz)



Mode 4 (5.06 Hz)



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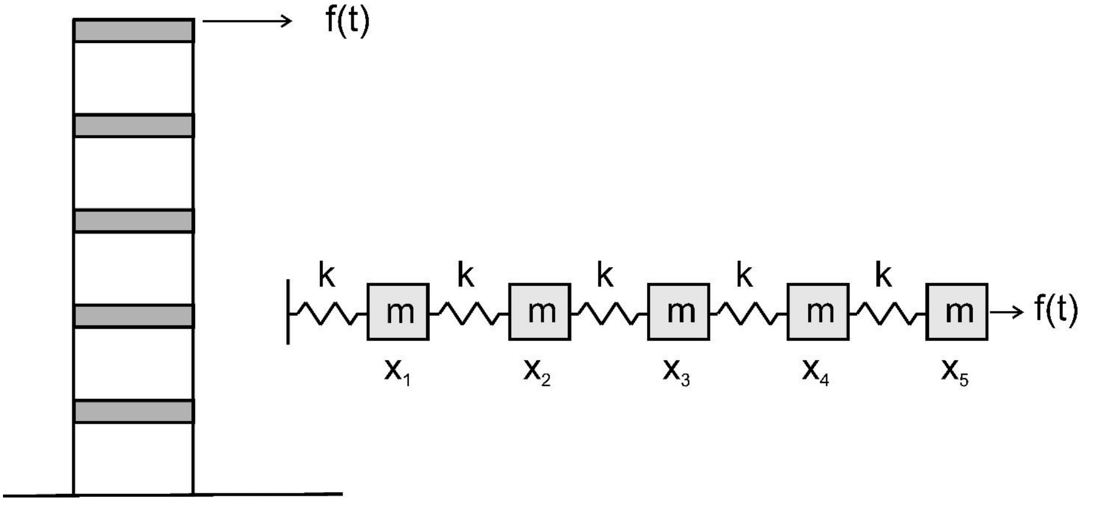
5 storey building



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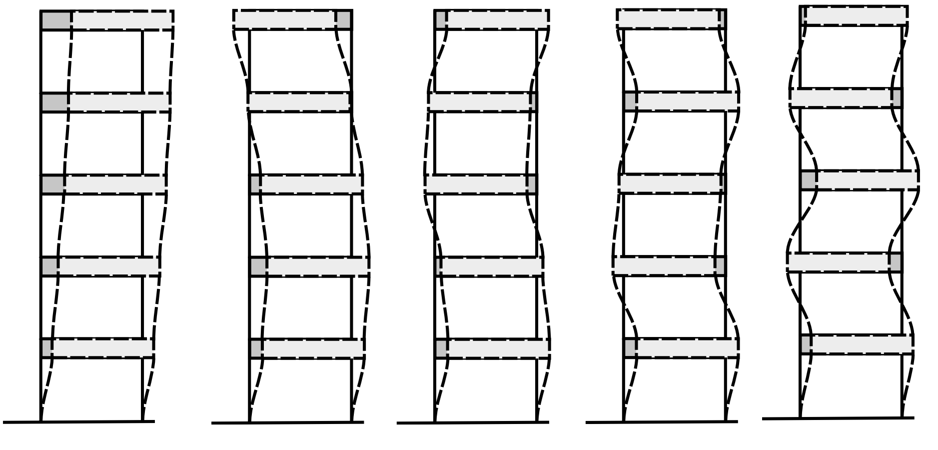
5 storey building

$$k = 27000 kN/m$$
$$m = 2500 kg$$

Mode 1 (4.05 Hz) Mode 2 (11.82 Hz) Mode 3 (18.64 Hz) Mode 4 (23.95 Hz) Mode 5 (27.31 Hz)



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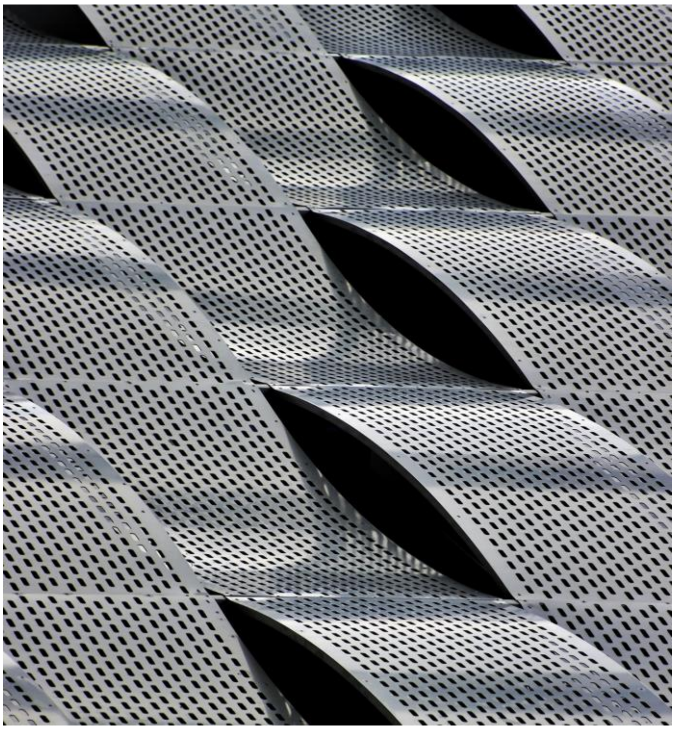
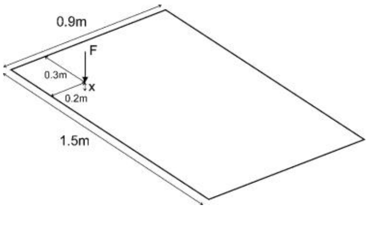
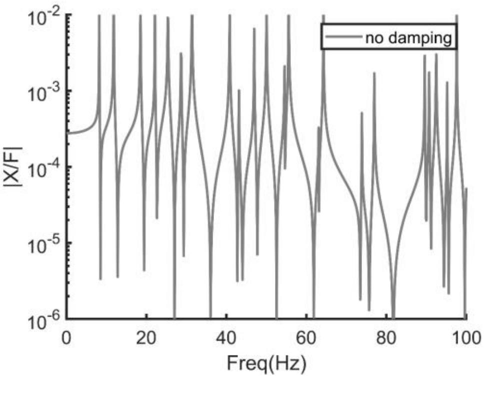
RESPONSE TO DYNAMIC FORCES



Harmonic response of the cantilever plate



$$(K - \omega^2 M)X = F$$



$$X(\omega) = \sum_{i=1}^{n} \frac{\psi_i^T F \psi_i}{\mu_i(\omega_i^2 - \omega^2)}$$ n=25

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Global damping models

For proportional (global) damping models

$$\mu_i \ddot{z}_i + 2\xi_i \mu_i \omega_i \dot{z}_i + \mu_i \omega_i^2 z_i = F_i$$

Rayleigh damping $\Rightarrow \xi_i = \frac{1}{2}\left(\alpha\omega_i + \frac{\beta}{\omega_i}\right)$

Loss factor $\Rightarrow \xi_i = \frac{\eta}{2}$ Constant modal damping

Modal damping $\Rightarrow \xi_i$ For each mode

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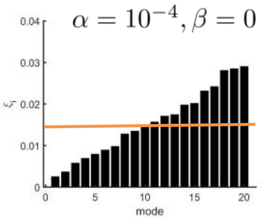
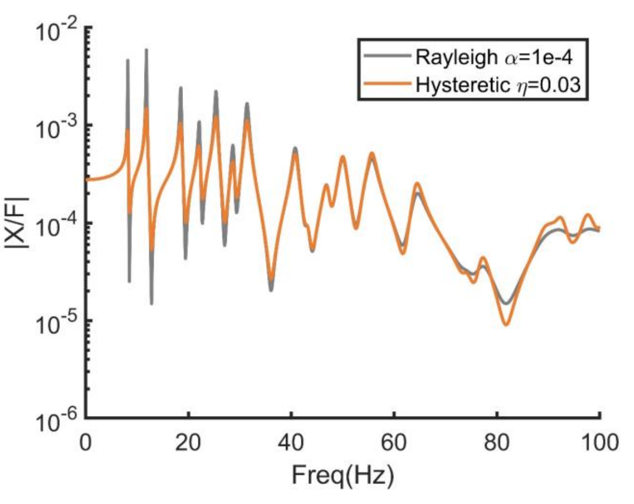
Harmonic response with damping

$\alpha = 10^{-4}, \beta = 0$



$\eta = 0.03 \Rightarrow \xi_i = 0.015$



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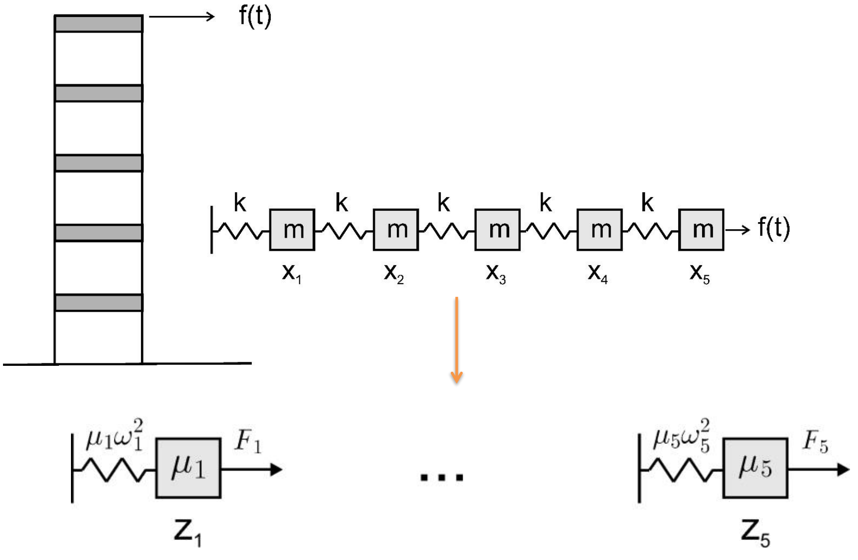
Response of a building excited by a point force



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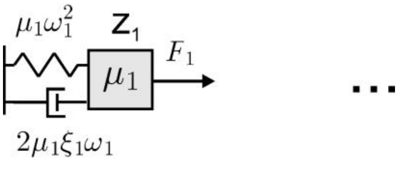
Damping model

$$x(t) = \sum_{i=1}^{n} z_i(t)\psi_i$$



$$\mu_i\ddot{z}_i + 2\mu_i\xi_i\omega_i\dot{z}_i + \mu_i\omega_i^2 z_i = F_i$$

Frequency response function

$$X(\omega) = \sum_{j=1}^{n} \frac{\psi_j^T F\psi_j}{\mu_j(\omega_j^2 - \omega^2 + 2i\xi_j\omega\omega_j)}$$

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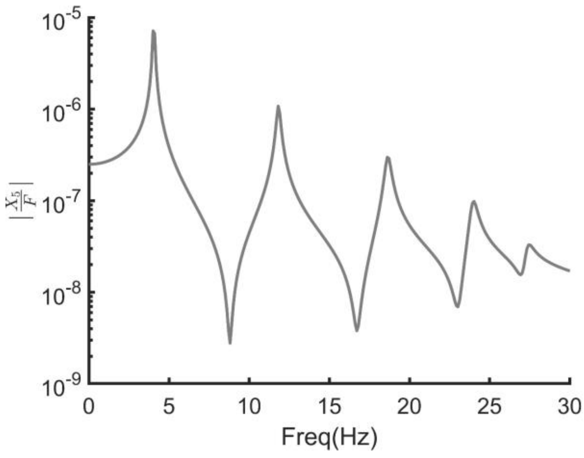
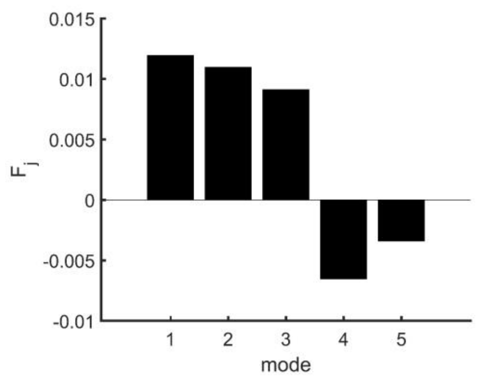
Response of a building excited by a point force

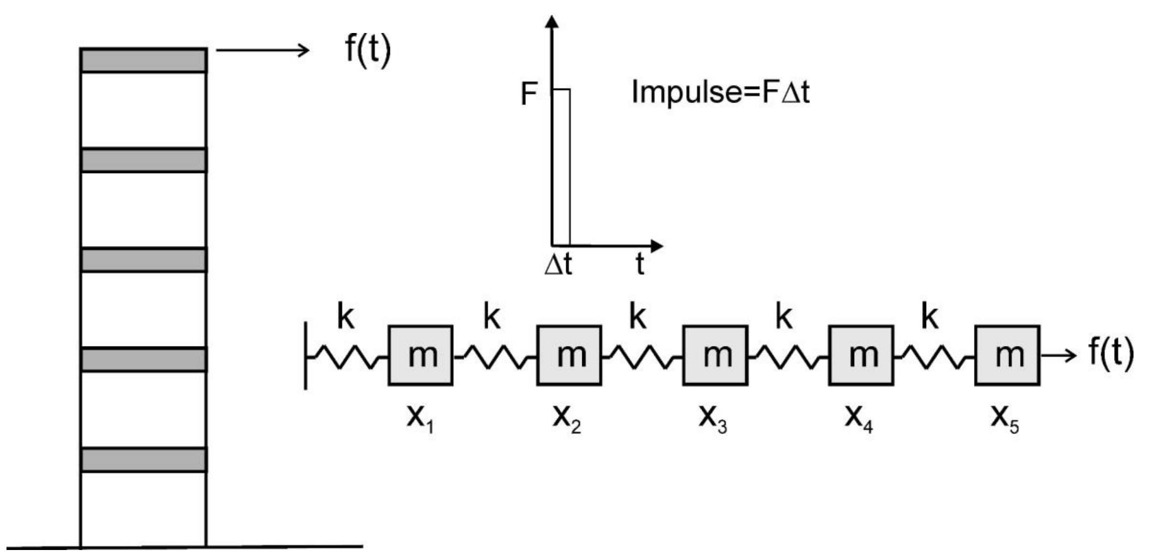
$$X(\omega) = \sum_{j=1}^{n} \frac{\psi_j^T F \psi_j}{\mu_j(\omega_j^2 - \omega^2 + 2i\xi_j\omega\omega_j)} \qquad \xi_i = 0.01 \quad i = 1...5$$



$$F_j = \psi_j^T F$$

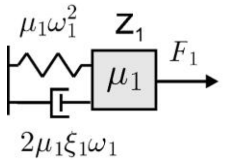


Impulse response of a building



Impulse response of a building

$$x(t) = \sum_{i=1}^{n} z_i(t)\psi_i$$



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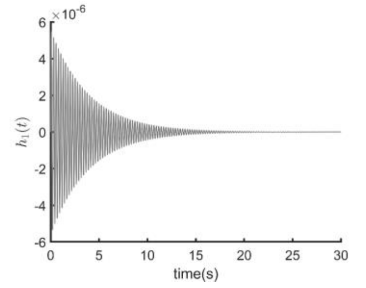
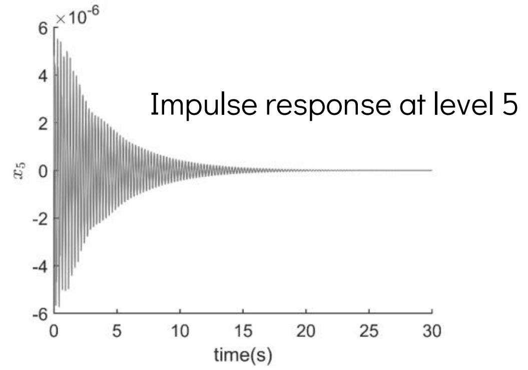


$$z_i(t) = F_i h_i(t) = F_i \frac{e^{-\xi_i\omega_i t}}{\mu_i\omega_{di}} \sin(\omega_{di} t)$$



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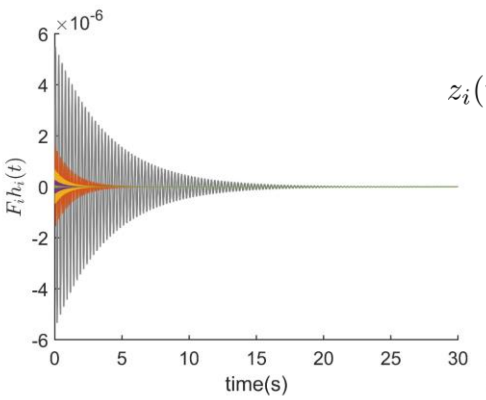
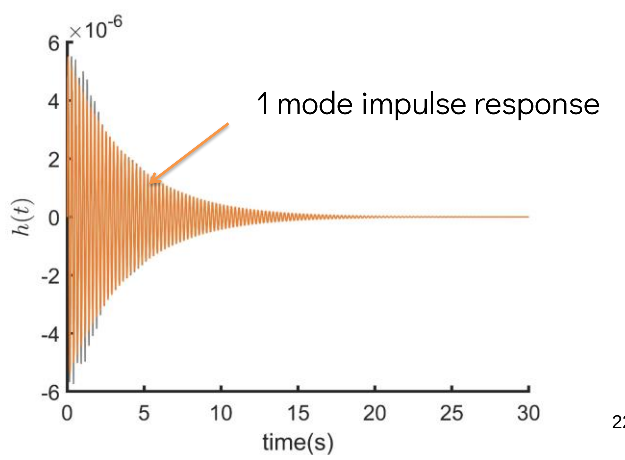
Impulse response of a building



$$z_i(t) = F_i h_i(t) = F_i \frac{e^{-\xi_i\omega_i t}}{\mu_i\omega_{di}} \sin(\omega_{di} t)$$



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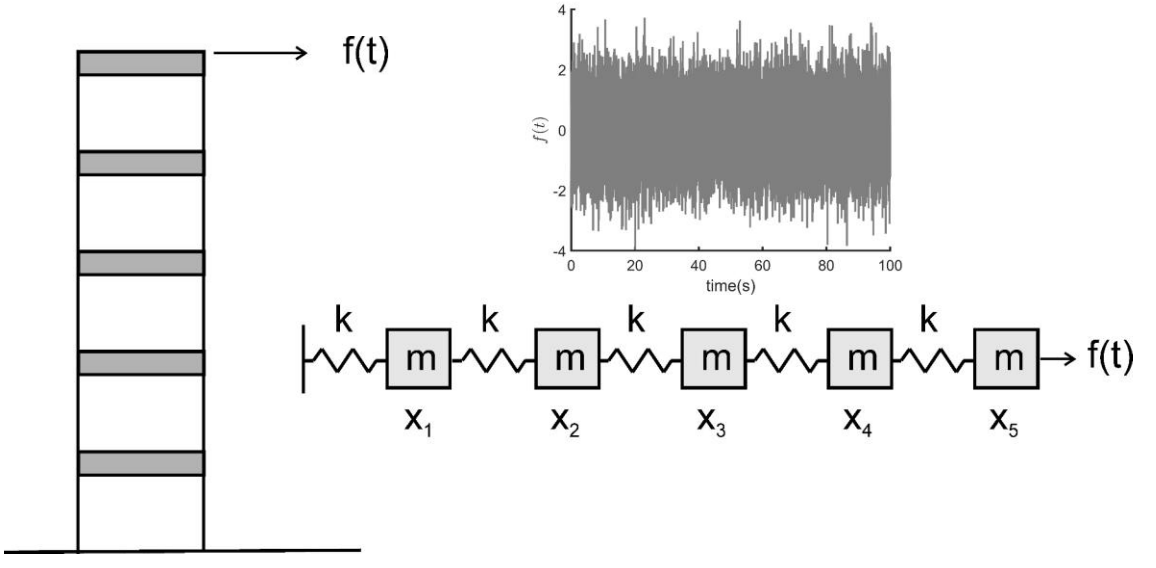
Random force excitation



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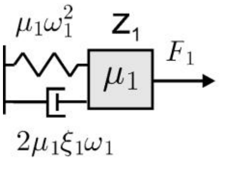
Random force excitation

$$x(t) = \sum_{i=1}^{n} z_i(t)\psi_i$$

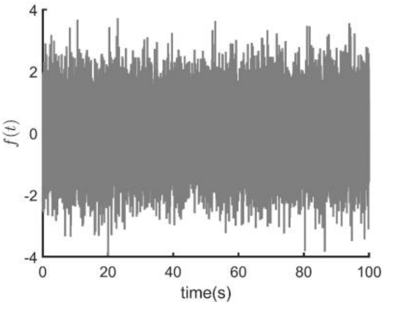


...

$$\mu_i \ddot{z}_i + 2\mu_i\xi_i\omega_i\dot{z}_i + \mu_i\omega_i^2 z_i = F_i$$

$$F_i(t) = \psi_i^T f(t)$$

Modal excitation



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Random force excitation

$$\mu_i \ddot{z}_i + 2\mu_i \xi_i \omega_i \dot{z}_i + \mu_i \omega_i^2 z_i = F_i \qquad F_i(t) = \psi_i^T f(t)$$

Impulse response

$$h_i(t) = \frac{e^{-\xi_i \omega_i t}}{\mu_i \omega_{di}} \sin(\omega_{di} t) \qquad \omega_{di} = \omega_i \sqrt{1 - \xi_i^2}$$

Convolution

$$z_i(t) = h_i(t) * F_i(t)$$

Expansion

$$x(t) = \sum_{i=1}^{n} z_i(t) \psi_i$$

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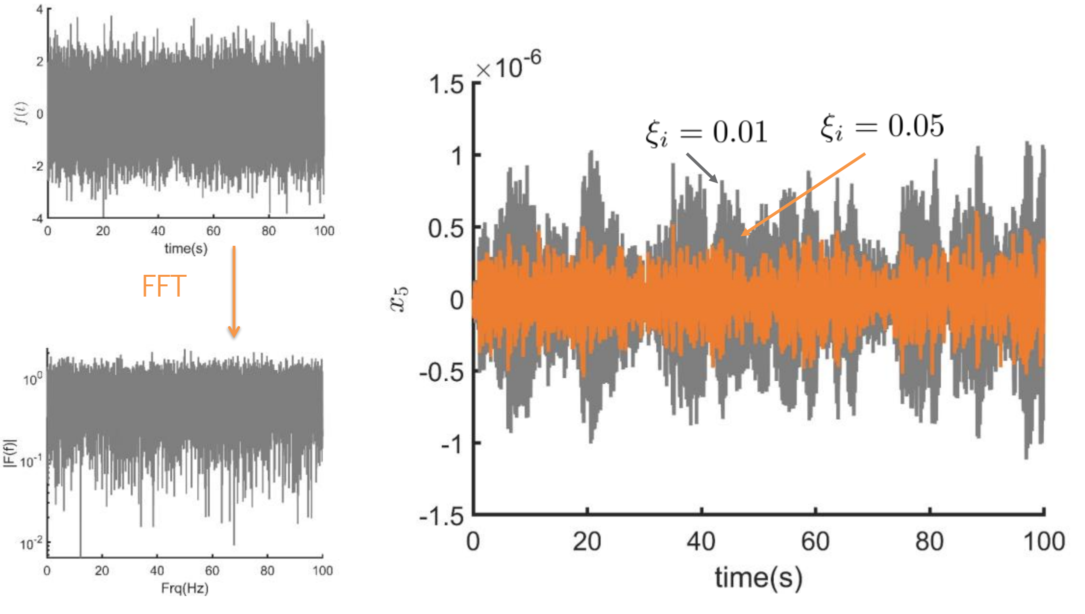
Random force excitation



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RESPONSE TO BASE ACCELERATION

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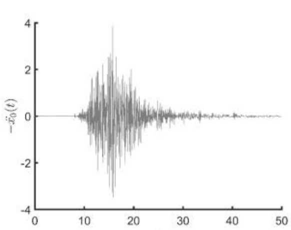
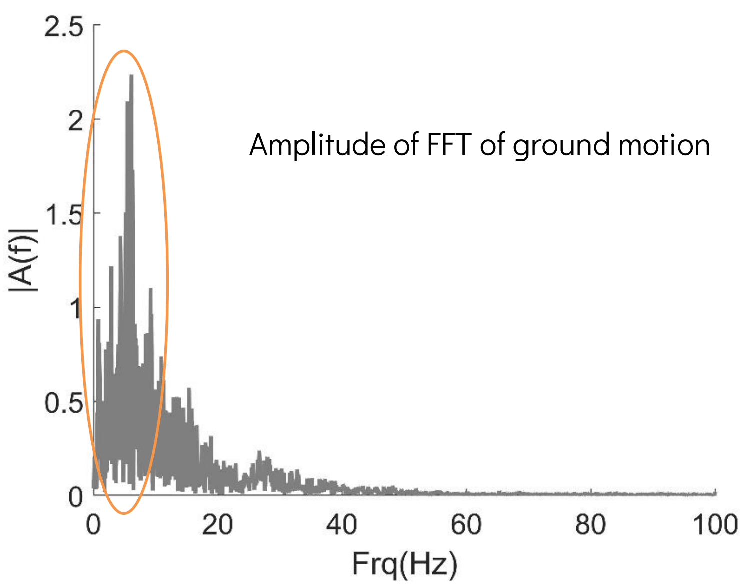
Frequency analysis of the source



Ground motion (acceleration)



Amplitude of FFT of ground motion

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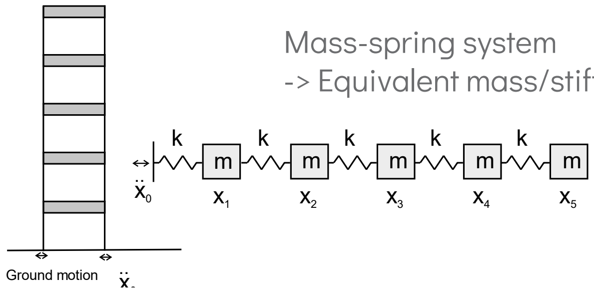
High-rise building model



Mass-spring system
-> Equivalent mass/stiffness

Finite element model

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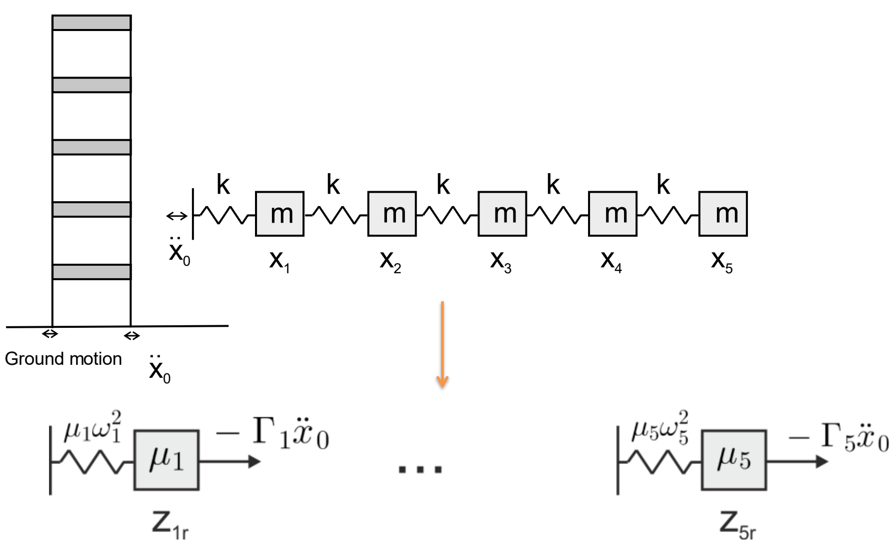
High-rise building model



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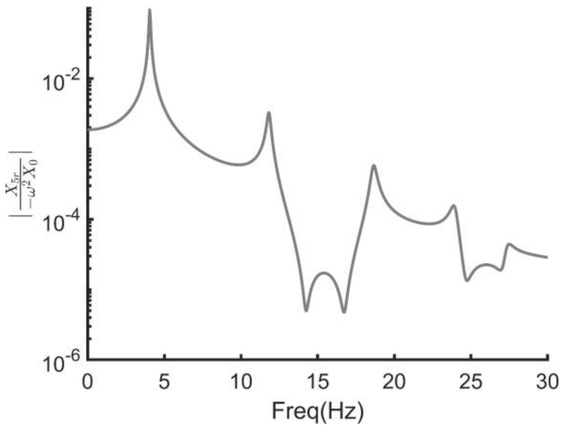
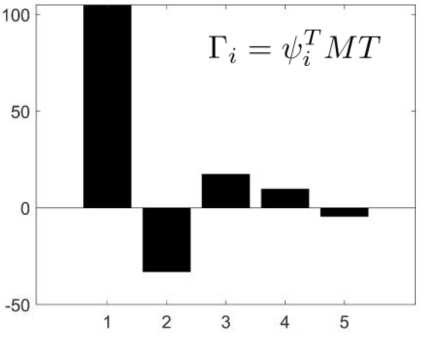
High-rise building model : base excitation FRF

$$X(\omega) = \sum_{j=1}^{n} \frac{-\Gamma_j \ddot{x}_0 \psi_j}{\mu_j(\omega_j^2 - \omega^2 + 2i\xi_j\omega\omega_j)}$$

$\xi_i = 0.01 \quad i = 1...5$





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High-rise building model : response to earthquake

$$M\ddot{x}_r + C\dot{x}_r + Kx_r = -M\ddot{x}_b \qquad \ddot{x}_b = T\,\ddot{x}_0$$

$$\longrightarrow \quad M\ddot{x}_r + C\dot{x}_r + Kx_r = -M\,T\ddot{x}_0$$

Projection in the modal basis using mode shapes with <u>fixed base</u> ($x_b = 0$)

$$x_r = \sum_{i=1}^{n} z_{ri}(t)\psi_i \qquad x_r = \Psi z_r$$

$$\Psi^T M\Psi\ddot{z}_r + \Psi^T C\Psi\dot{z}_r + \Psi^T K\Psi z_r = -\Psi^T MT\ddot{x}_0$$

$$\boxed{\mu_i\ddot{z}_{ri} + 2\mu_i\xi_i\omega_i\dot{z}_{ri} + \mu_i\omega_i^2 z_{ri} = -\Gamma_i\ddot{x}_0}$$

$\Gamma_i = \psi_i^T MT$ Modal acceleration factor

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Base excitation : time domain response

$$\mu_i \ddot{z_{ri}} + 2\mu_i \xi_i \omega_i \dot{z_{ri}} + \mu_i \omega_i^2 z_{ri} = -\Gamma_i \ddot{x}_0 \qquad \Gamma_i = \psi_i^T M T$$

Impulse response

$$h_i(t) = \frac{e^{-\xi_i \omega_i t}}{\mu_i \omega_{di}} \sin(\omega_{di} t) \qquad \omega_{di} = \omega_i \sqrt{1 - \xi_i^2}$$

Convolution

$$z_{ri}(t) = h_i(t) * -\Gamma_i \ddot{x}_0$$

Expansion

$$x_r(t) = \sum_{i=1}^{n} z_{ir}(t) \psi_i$$

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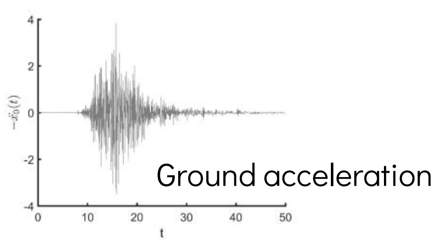
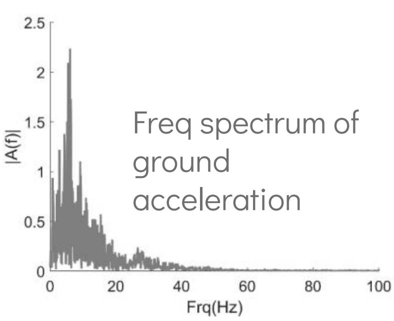
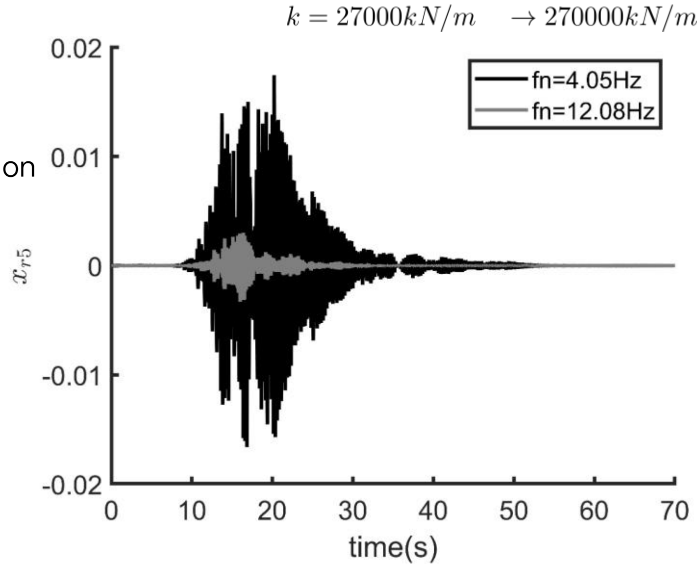
Effect of a stiffness change





$k = 27000 kN/m \quad \rightarrow 270000 kN/m$



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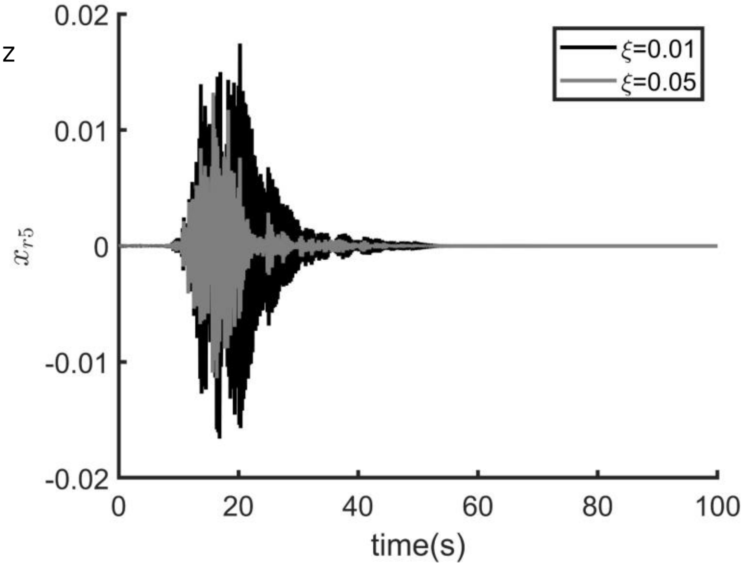
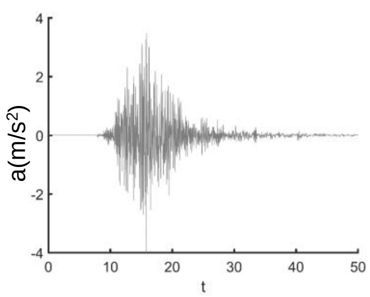
Adding damping

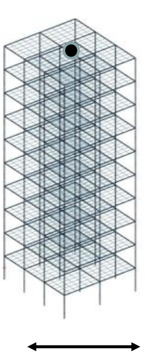
f_n=4.05 Hz



Earthquake response of a 10 storey building

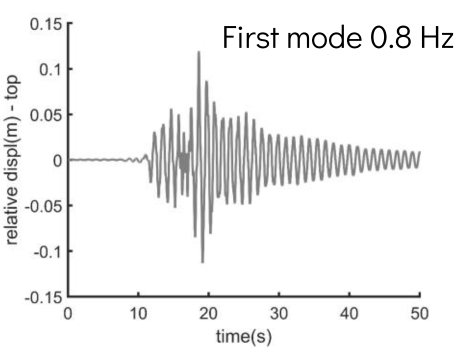
Modal damping / 20 modes (0-15 Hz)
$\xi = 0.01$





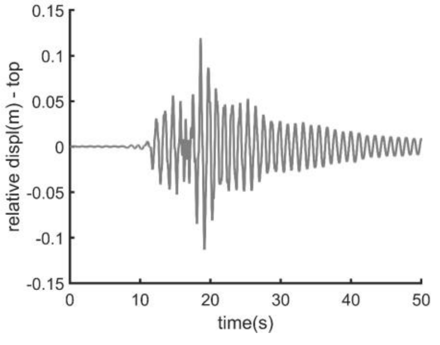
Ground motion (acceleration)

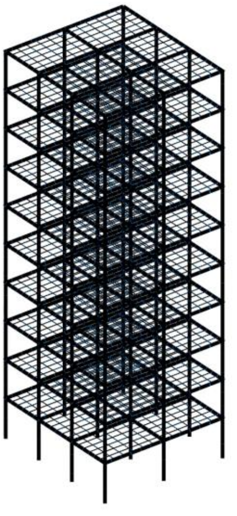
First mode 0.8 Hz



Earthquake response of a 10 storey building

t=0





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Summary

Excitation
Freq
Time

→

Model
M,C,K
Modal properties

→

Response
Freq
Time

↓

Acceptable ?

Yes

No →

New design
Remedial measures

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