FLOW INDUCED VIBRATIONS

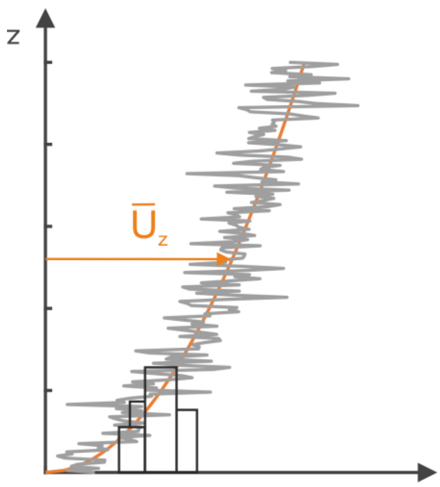
1

CONSTANT FLOW AND TURBULENCE

2

Wind excitation profile

Constant wind and turbulence



Wind forces :

$$f_{tot}(t) = \frac{1}{2}\rho C_d \Omega \left[U + u(t)\right]^2$$

$$\simeq \underbrace{\frac{1}{2}\rho C_d \Omega U^2}_{f_{avg}} + \underbrace{\rho C_d \Omega U u(t)}_{f_{turb}(t)}$$

Aerodynamic forces

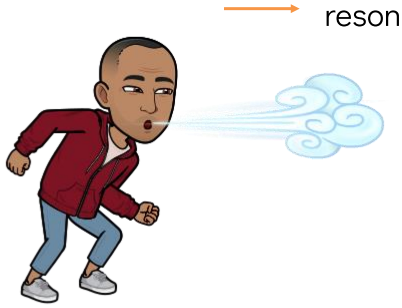
Constant flow

- Constant force
- Dynamic vortex excitation
- Self-excited vibrations
  - Galopping
  - Divergence
  - Flutter

Turbulent flow

- Dynamic force

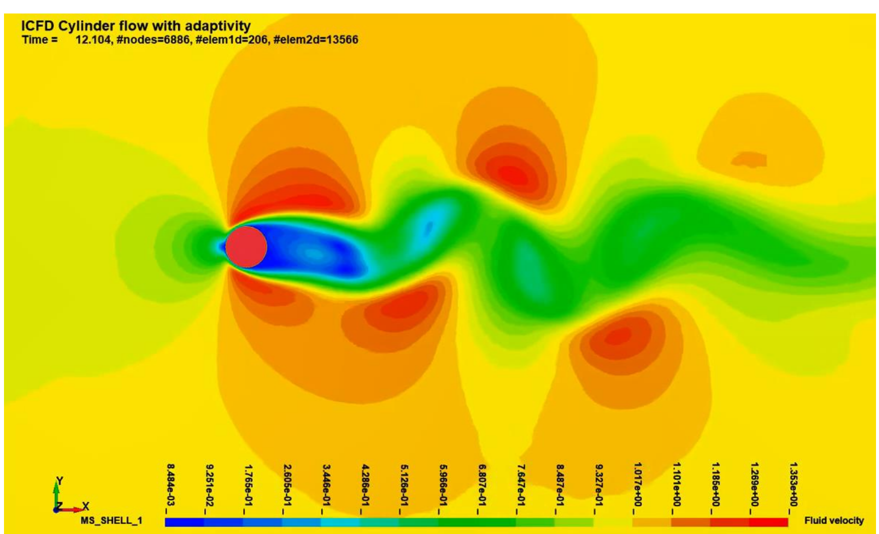
→ resonance



VORTEX INDUCED VIBRATIONS

5

Vortex induced vibrations

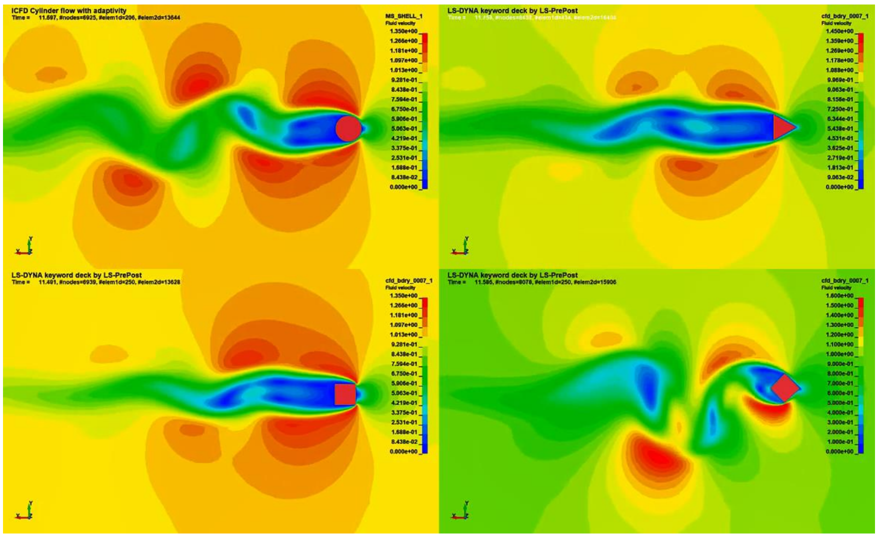


https://youtu.be/mPvTz7KqCiA

6

6

Vortex induced vibrations

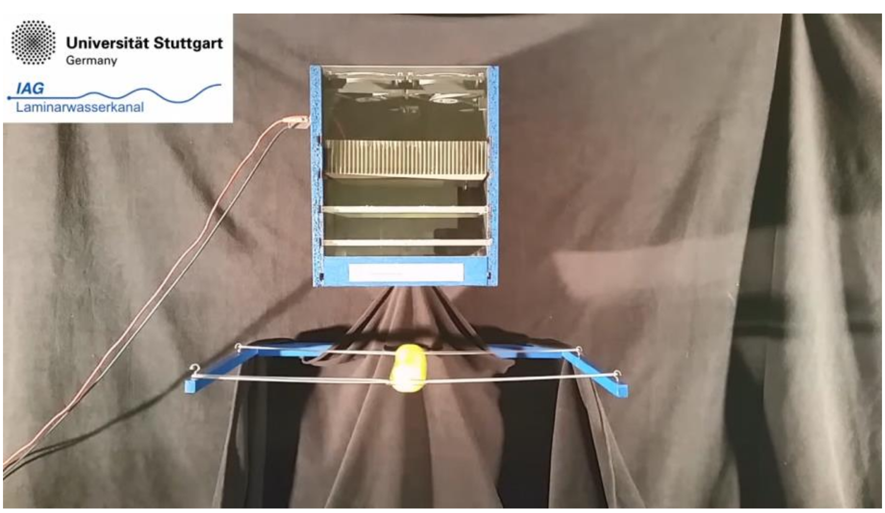


https://youtu.be/mPvTz7KqCiA

7

7

Vortex induced vibrations

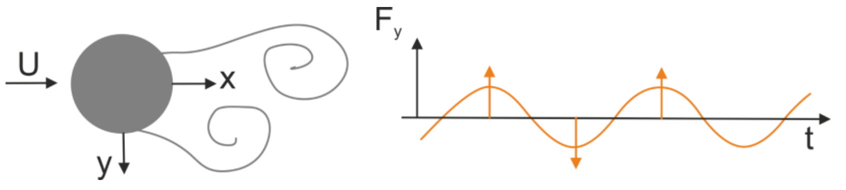


https://youtu.be/KHVy049pouE

8

8

Vortex induced vibrations

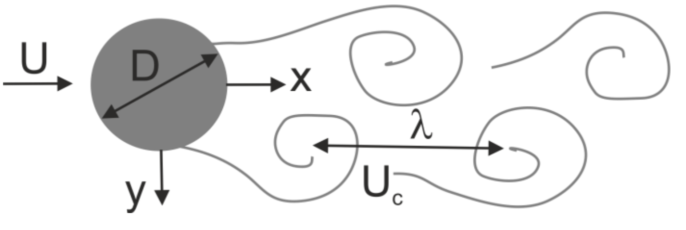


- Alterning vortices induce a sinusoïdal force to the cylinder
- The frequency is related to the wind speed

$$f_v = \frac{S_t}{D} U$$

9

The Strouhal Number



Strouhal Number :

$$S_t = f_v \frac{D}{U} = \frac{U_c}{U} \frac{D}{\lambda}$$

$$U_c = \lambda f_v$$

For a cylinder

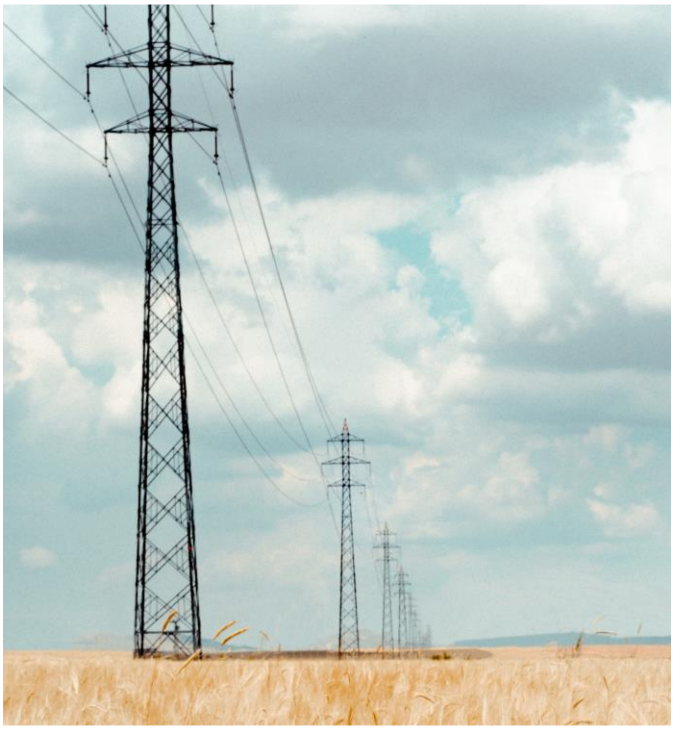
$$\frac{U_c}{U} = 0.5$$

$$\frac{D}{\lambda} = 0.4$$

$\longrightarrow S_t = 0.2$

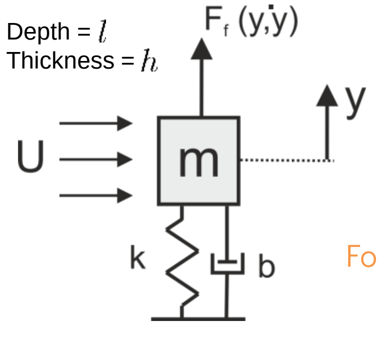
10

SELF-EXCITED VIBRATIONS



11

Self-excited vibrations



$$ml\ddot{y} + b\dot{y} + ky = F_f(\dot{y}, y)$$

Force applied by the fluid:

$$F_f(\dot{y}, y) = \frac{1}{2}\rho U^2 hl C_{f_1}(\dot{y}, U) + \frac{1}{2}\rho U^2 hl C_{f_2}(y, U)$$

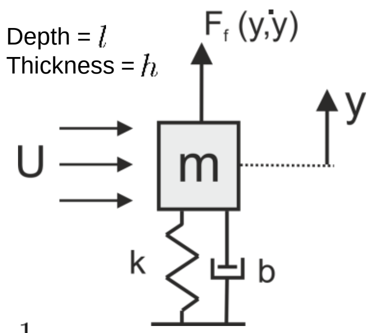
Aerodynamic coefficient depending on velocity of elastic body

Aerodynamic coefficient depending on displacement of elastic body

12

12

Self-excited vibrations



$$ml\ddot{y} + b\dot{y} + ky = F_f(\dot{y}, y)$$

$$\eta = y/l, \quad \tau = \omega_n t, \quad U_r = U/(\omega_n h), \quad \mu_r = \rho h^2/m$$

$$\ddot{\eta} + 2\xi\dot{\eta} + \eta = \frac{1}{2}\mu_r U_r^2 C_{f_1}(\dot{\eta}, U_r) + \frac{1}{2}\mu_r U_r^2 C_{f_2}(\eta, U_r)$$

Third order aeroelastic coefficients :

$$\frac{1}{2}\mu_r U_r^2 C_{f_1}(\dot{\eta}, U_r) = \beta_1(U_r)\dot{\eta} - \beta_3(U_r)\dot{\eta}^3 \qquad \frac{1}{2}\mu_r U_r^2 C_{f_2}(\eta, U_r) = \gamma_1(U_r)\eta - \gamma_3(U_r)\eta^3$$

$$\ddot{\eta} + \left[2\xi - \beta_1(U_r) + \beta_3(U_r)\dot{\eta}^2\right]\dot{\eta} + \left[1 - \gamma_1(U_r) + \gamma_3(U_r)\eta^2\right]\eta = 0$$

Damping varies with
- Fluid velocity
- Amplitude of elastic body velocity

Stiffness varies with
- Fluid velocity
- Amplitude of elastic body displacement

13

13

The Pole-Residue model

The frequency response function of a one dof system is :

$$H(\omega) = \frac{X(\omega)}{F(\omega)} = \frac{1}{m}\frac{1}{\omega_n^2 - \omega^2 + 2j\xi\omega\omega_n}$$

The Pole-residue model in the frequency domain is :

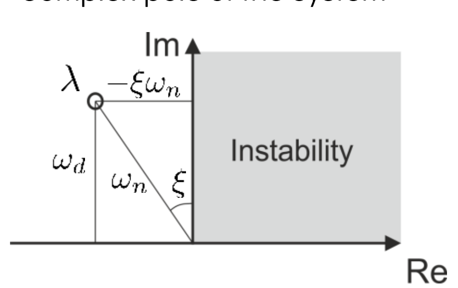
$$H(\omega) = \frac{R}{j\omega - \lambda} + \frac{R^*}{j\omega - \lambda^*}$$

$$\lambda = -\xi\omega_n + j\omega_d$$

$$R = -\frac{j}{2m\omega_d}$$

$$\omega_d = \omega_n\sqrt{1 - \xi^2}$$

Complex pole of the system

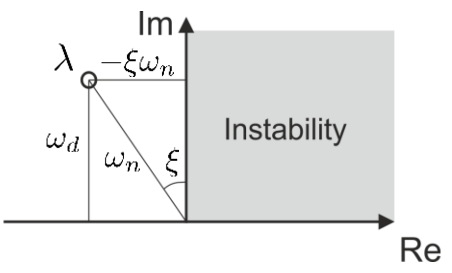


14

14

Complex poles and stability

Complex pole of the system



One DOF system impulse response

$$h(t) = \frac{e^{-\xi\omega_n t}}{m\omega_d} sin(\omega_d t)$$

Pole-residue model in the time domain is :

$$h(t) = Re^{\lambda t} + R^* e^{\lambda^* t}$$

$$\lambda = -\xi\omega_n + j\omega_d$$

Instability occurs when pole crosses the imaginary axis (negative damping)

15

15

Self-excited vibrations

$$\ddot{\eta} + \left[2\xi - \beta_1(U_r) + \beta_3(U_r)\dot{\eta}^2\right]\dot{\eta} + \left[1 - \gamma_1(U_r) + \gamma_3(U_r)\eta^2\right]\eta$$

For small amplitudes :

- Galopping occurs when $2\xi - \beta_1 = 0$ ⟶ Oscillatory instability.
  Galopping can occur in translation or in rotation, in the later case it is commonly referred to as rotational flutter
- Divergence occurs when $1 - \gamma_1 = 0$ ⟶ Static instability

For large amplitudes :
In general, coefficients β_3, γ_3 limit the amplitude of motion
⟶ Limit cycle oscillations

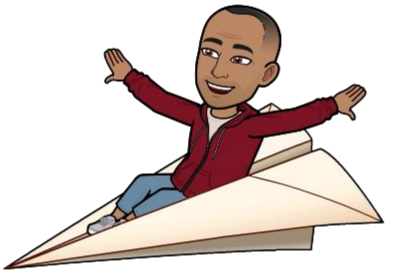
16

16

Self-excited vibrations

Main mechanism :

- Fluid-structure interaction changes effective stiffness/damping
- Static or dynamic instability can occur at certain wind speeds
- As every structure is subject to small vibration levels, instability leads to magnification of the existing vibrations which can cause
  - Fatigue damage (over the long term)
  - Failure of the structure (very rapidly)



17

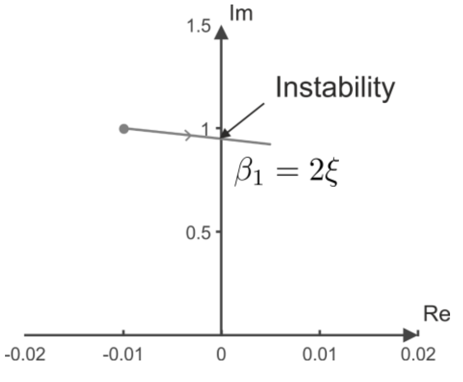
17

Linear model and stability

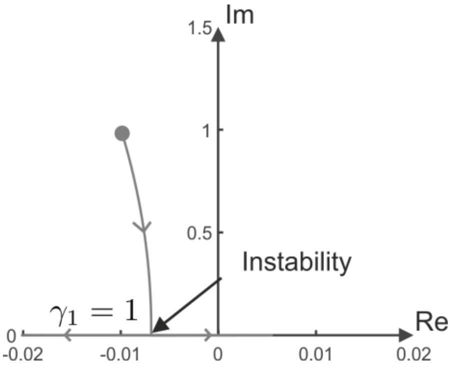
$$\ddot{\eta} + [2\xi - \beta_1(U_r)]\,\dot{\eta} + [1 - \gamma_1(U_r)]\,\eta = 0$$

Variable damping   Variable stiffness

Galopping (oscillatory)



Divergence (static)

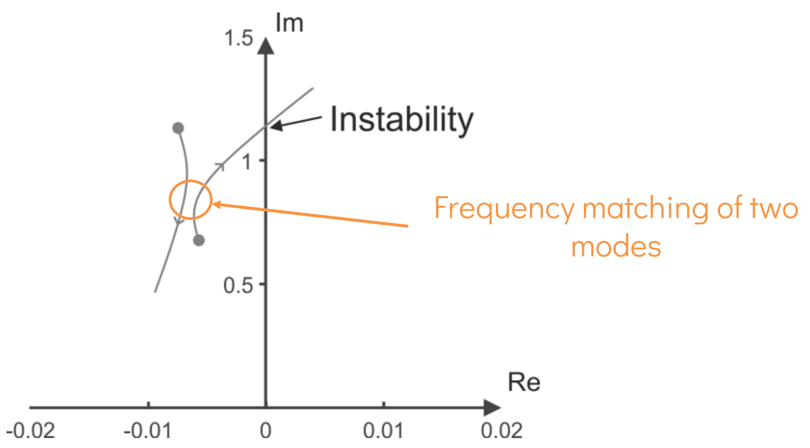


18

18

Stability for MDOF systems

- Divergence/galopping of a single mode
- Coupled mode flutter



19

19

Aerodynamic forces

Constant flow

- Constant force
- Dynamic vortex excitation
- Self-excited vibrations
  - Galopping
  - Divergence
  - Flutter

Turbulent flow

- Dynamic force

→ resonance

20

20