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Footbridge in Durbuy



From [Bureau Greisch, 2020]

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Vibration source

Representative types of activity		
Designation	Definition	Design activity rate [Hz]
"walking"	walking, continuous ground contact	1.6 – 2.4
"running"	running, discontinuous ground contact	2.0 – 3.5

Representative types of activity	Activity rate [Hz]	Fourier coefficient and phase lag					
		α_1	ϕ_1	α_2	ϕ_2	α_3	ϕ_3
"walking"	vertical 2.0	0.4		0.1	$\pi/2$	0.1	$\pi/2$
	2.4	0.5					
	forward 2.0	$0.5(\alpha_{1/2}=0.1)$		0.2			
	lateral 2.0	$\alpha_{1/2} = 0.1$		$\alpha_{3/2} = 0.1$			
"running"	2.0 3.0	1.6		0.7		0.2	

From ‘Vibration problems in structures’, H. Bachman, 1995

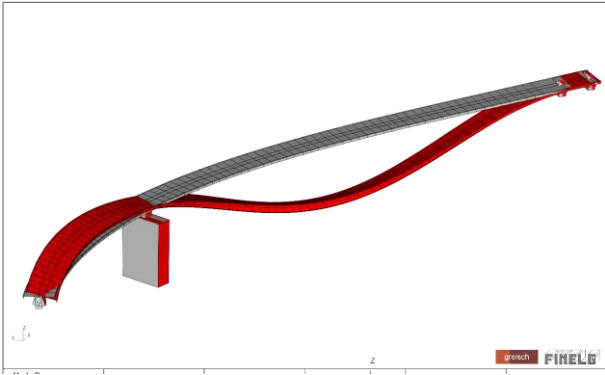
→ Main frequencies of excitation from 1.5 to 3 Hz
! Harmonics !

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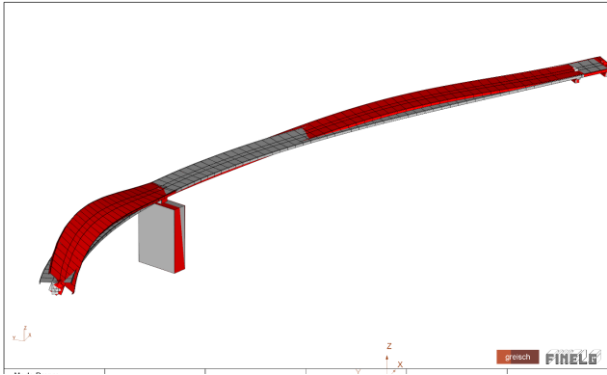
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Footbridge in Durbuy

Mode 1 (2 Hz)



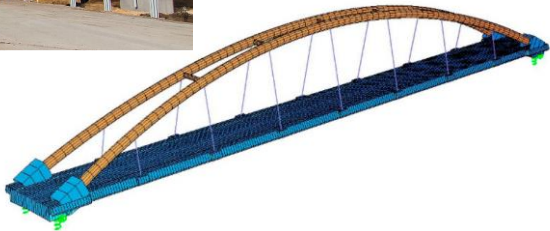
Mode 2 (4.16 Hz)



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Arch footbridge S8 expressway – Poland



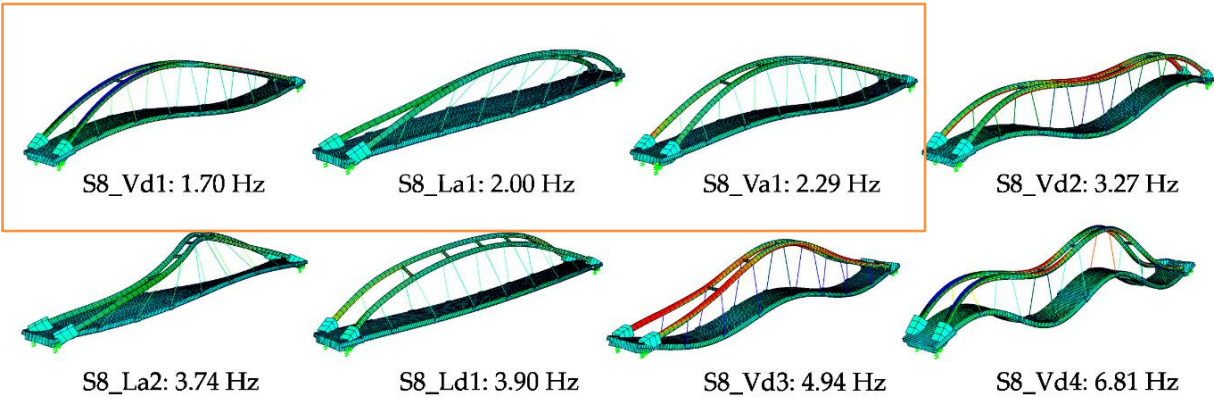
[A. Banas, R. Jankowski *Experimental and Numerical Study on Dynamics of Two Footbridges with Different Shapes of Girders*, Appl. Sci. 2020, 10, 4505; doi:10.3390/app10134505]

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Mode shapes

Potential resonance (first harmonic)



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Pedestrian induced vibrations



<https://www.youtube.com/watch?v=zpGZN8r2QxY>

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Pedestrian induced vibrations



<https://www.youtube.com/watch?v=gQK21572oSU>

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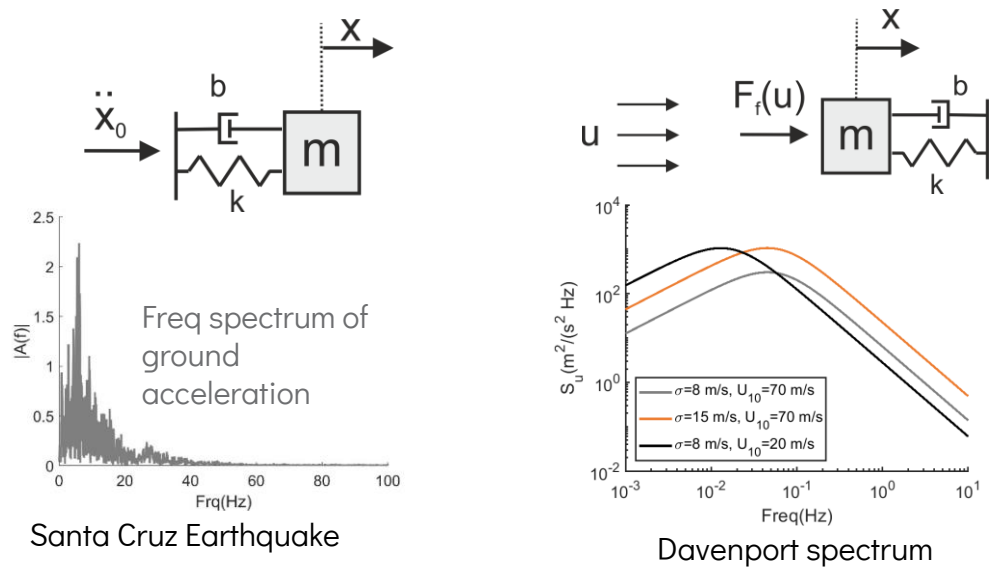
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HIGH-RISE BUILDINGS



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Wind and earthquake excitations



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Wind excitation



303 m high-rise building, Guangzhou China

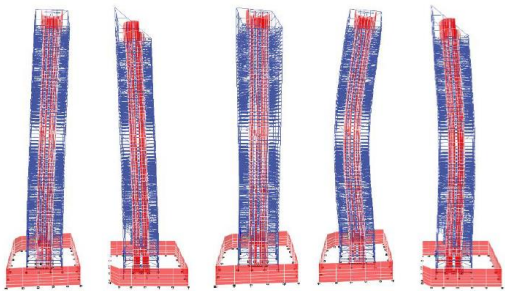


Figure 16. Mode shapes for the first five orders from simulation via finite element method.

Table 6. Results of natural frequencies of Leatop Plaza.

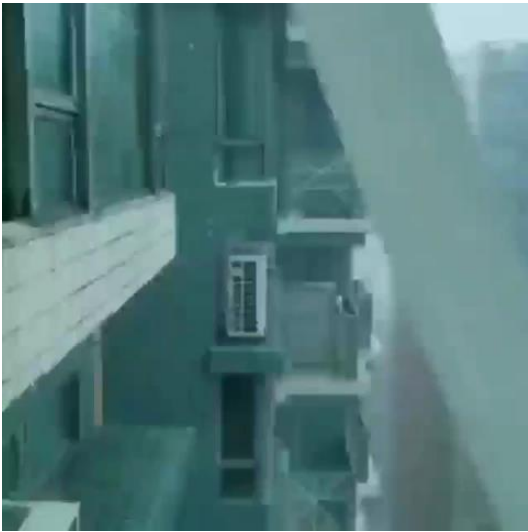
Mode No.	Measurement (Hz)	Simulation (Hz)	Difference (%)	Mode Type
1	0.183	0.164	10.4	1st mode in X direction (sway)
2	0.182	0.167	8.24	1st mode in Y direction (sway)
3	0.429	0.388	9.56	1st mode in Z direction (torsion)
4	0.683	0.592	13.3	2nd mode in X direction (sway)
5	0.656	0.612	6.71	2nd mode in Y direction (sway)

Zhi Li, J. Fu, Y. He, Z. Liu, J. Wu, R. Rao and C.T. Ng, *Structural Responses of a Supertall Building Subjected to a Severe Typhoon at Landfall*, Appl. Sci. 2020, 10, 2965

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Building swaying in the wind

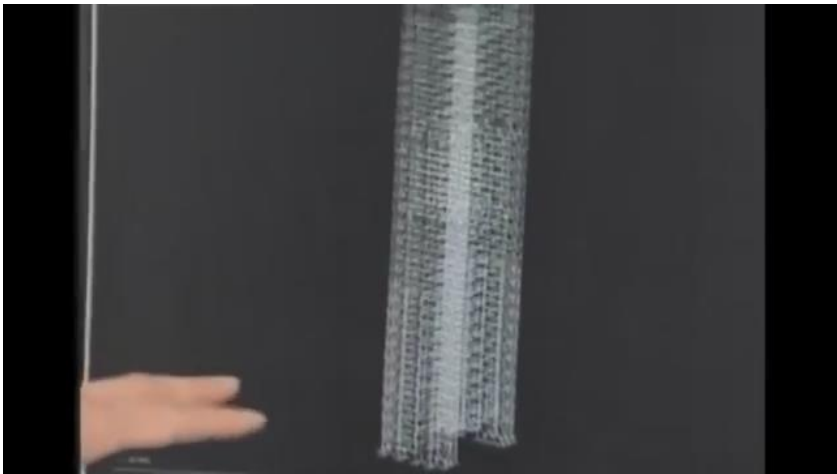


<https://www.youtube.com/watch?v=i1sHJRCJPS4>

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Building swaying after earthquake



<https://www.youtube.com/watch?v=2t2xxKMN-Ic&t=230s>

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MACHINE INDUCED VIBRATIONS

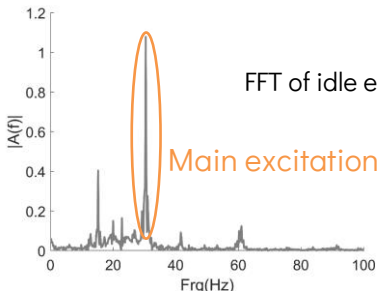


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Machine induced vibrations



Force transmitted to the surroundings



FFT of idle engine acceleration

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Washing machines



<https://www.youtube.com/watch?v=SRbFxgezAX0>

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Harvester



[Romaric BIAOU OLAYE]

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Tractor engine



<https://www.youtube.com/watch?v=7tJkI7dygfU>

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Helicopter ground resonance



<https://www.youtube.com/watch?v=ZcdYIkrQVzA>

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PRECISION EQUIPMENT
AND COMFORT

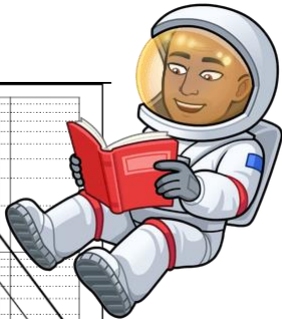
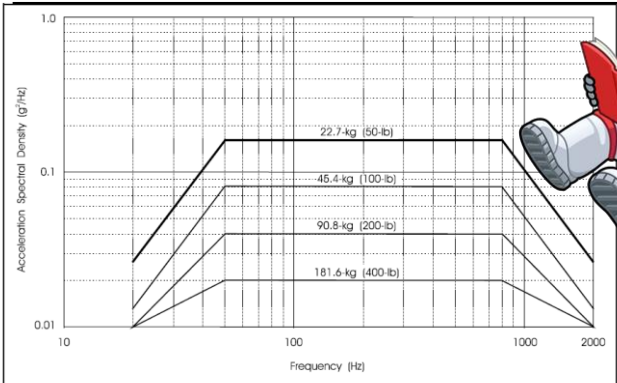
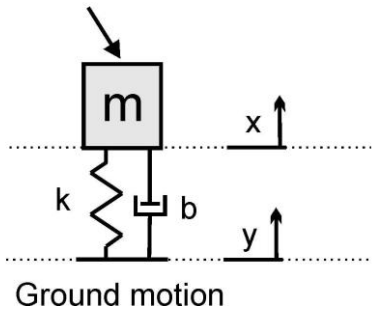


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Vibrations caused by the surroundings

Building,
Sensitive equipment

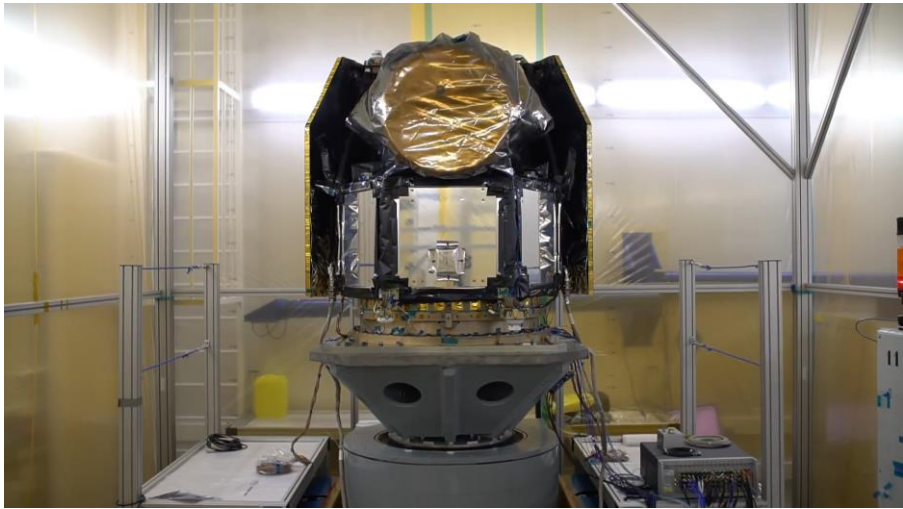


GSFC Standard (Nasa)

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Payload comfort in space launchers (satellites)



<https://www.youtube.com/watch?v=1O4PEqEXk5M>

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Comfort in sports cars

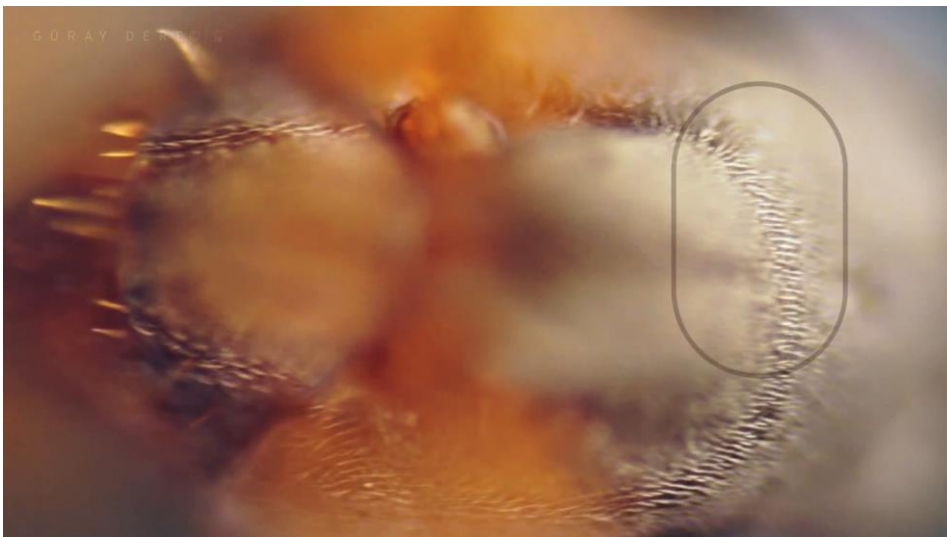


<https://www.youtube.com/watch?v=5bsuar1vo2g>

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Precision microscope vibration



<https://www.youtube.com/watch?v=9MFsymYmwLo>

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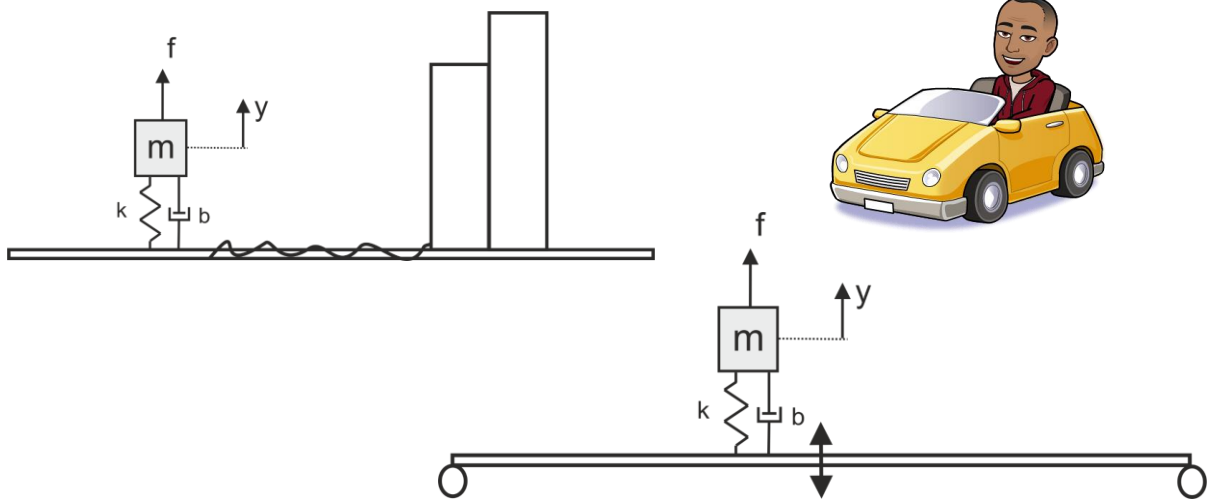
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VIBRATIONS CAUSED
BY TRAFFIC



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Vibrations caused by traffic



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Vibrations caused by traffic



<https://www.youtube.com/watch?v=ZAa6ehyIZ94>

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Vibrations caused by traffic

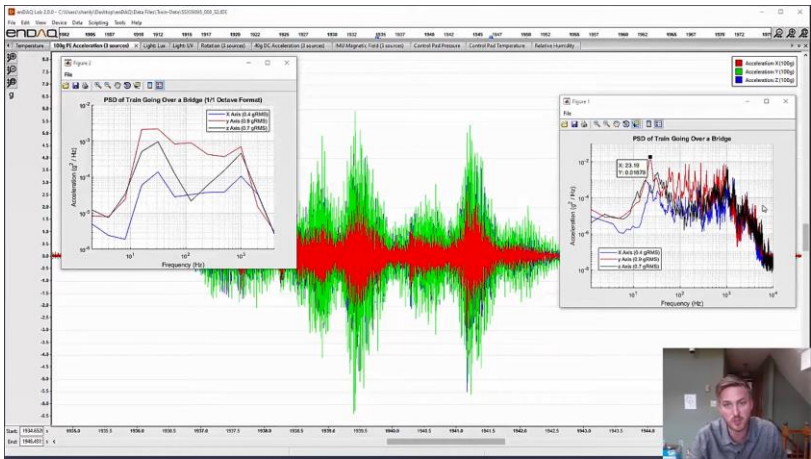


<https://www.youtube.com/watch?v=ioSyeKusvE8>

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Vibrations caused by traffic



<https://www.youtube.com/watch?v=ioSyeKusvE8>

30

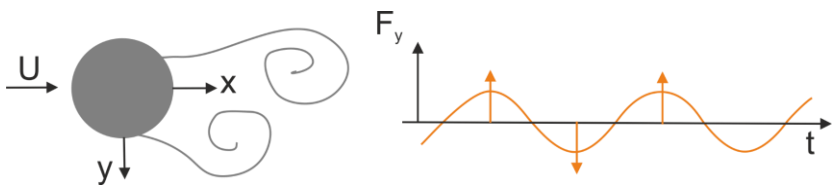
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VORTEX INDUCED VIBRATIONS



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Vortex Induced Vibrations



$$f_v = \frac{S_t}{D} U$$

Resonance $f_v = f_n$

Critical wind speed

$$U_c = \frac{D}{S_t} f_n$$

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Vortex Induced Vibrations



https://www.youtube.com/watch?v=YbZE_dgAqkc



<https://www.youtube.com/watch?v=-JA6EfdDeck>



<https://www.youtube.com/watch?v=80ExYy28moc>

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Vortex Induced Vibrations

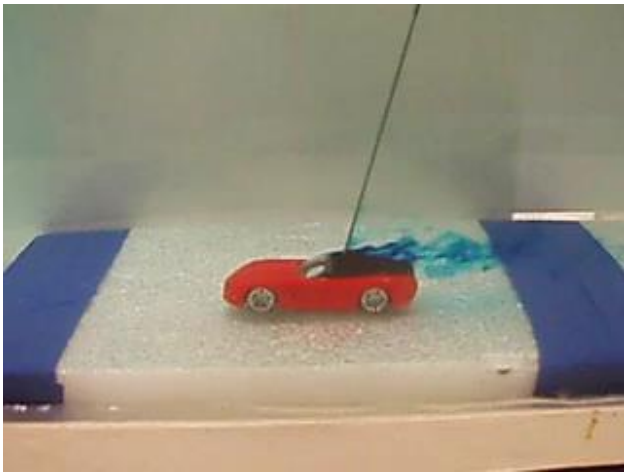


<https://www.youtube.com/watch?v=rlpUhgfEZPU>

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Vortex Induced Vibrations



<https://www.youtube.com/watch?v=oYegGVgvDb4>

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Vortex Induced Vibrations



<https://www.youtube.com/watch?v=qROefSbKcms>

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Vortex Induced Vibrations



<https://www.youtube.com/watch?v=J21uZjFVPak>

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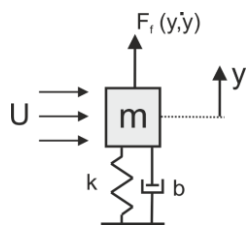
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GALOPPING AND DIVERGENCE



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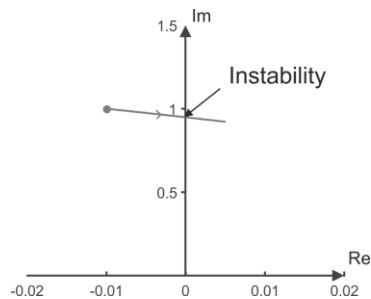
Galloping and divergence



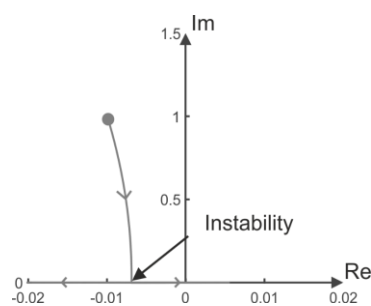
$$\ddot{\eta} + [2\xi - \beta_1(U_r)] \dot{\eta} + [1 - \gamma_1(U_r)] \eta = 0$$

Variable damping Variable stiffness

Galloping (oscillatory)



Divergence (static)



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Galloping



<https://www.youtube.com/watch?v=GEGbYRii1d4>

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Divergence



[From Vincent Denoël, ULiège]

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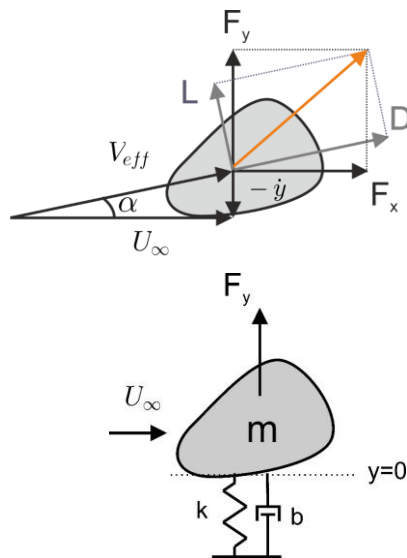
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FLUTTER



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Single mode flutter



$$F_y = L \cos \alpha + D \sin \alpha$$

$$\tan \alpha = \frac{-\dot{y}}{U_\infty}$$

$$m\ddot{y} + b\dot{y} + ky = F_y = L \cos \alpha + D \sin \alpha$$

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Single mode flutter

$$m\ddot{y} + b\dot{y} + ky = L \cos \alpha + D \sin \alpha$$

$$\simeq L + D\alpha$$

$$\simeq \left(L_0 + \frac{dL}{d\alpha}\alpha\right) + \left(D_0 + \frac{dD}{d\alpha}\alpha\right)\alpha$$

$$\simeq L_0 + \left(\frac{dL}{d\alpha} + D_0\right)\alpha$$

$$\simeq L_0 + \left(\frac{dL}{d\alpha} + D_0\right)\frac{-\dot{y}}{U_\infty}$$

$$m\ddot{y} + \left(b + \frac{1}{U_\infty}\left(\frac{dL}{d\alpha} + D_0\right)\right)\dot{y} + ky = 0$$

$$L = \frac{1}{2}tC_L\rho U_\infty^2$$

$$D = \frac{1}{2}tC_D\rho U_\infty^2$$

$$m\ddot{y} + \left(b + \frac{1}{2}t\rho\left(\frac{dC_L}{d\alpha} + C_D\right)U_\infty\right)\dot{y} + ky = 0$$

Can be negative and lead to zero damping at critical speed

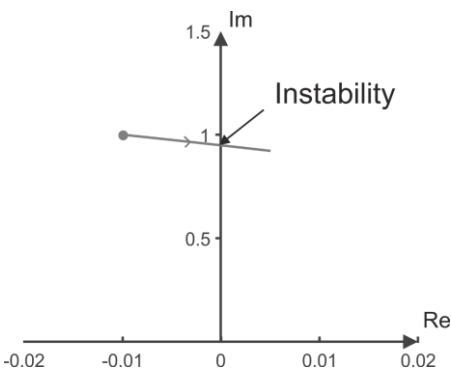
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Single mode flutter

$$m\ddot{y} + \left(b + \frac{1}{2}t\rho\left(\frac{dC_L}{d\alpha} + C_D\right)U_\infty \right) \dot{y} + ky = 0$$

Can be negative and lead to zero damping at critical speed

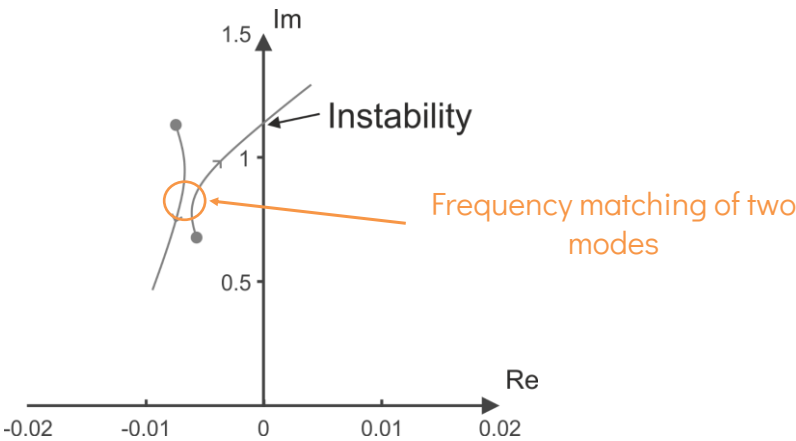


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Stability for MDOF systems

- Single mode flutter (galloping)
- Coupled mode flutter



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Flutter in Tacoma Narrows bridge



<https://www.youtube.com/watch?v=XggxeuFDaDU>

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Aeroelastic flutter in aircraft

Aeroelastic flutter in aircraft



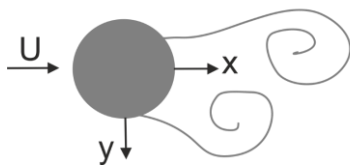
<https://www.youtube.com/watch?v=pEOmCkZyXzk>

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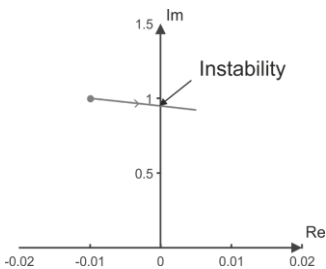
VIV vs instabilities

VIV



- Resonance
- Amplitude limited by damping

Galopping, flutter

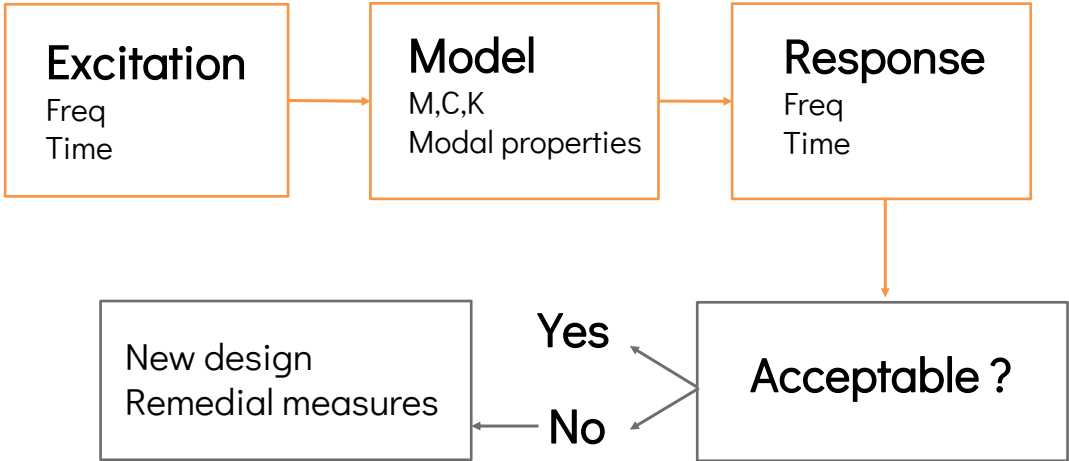


- Instability of a pole
- Unlimited amplitude (zero damping)

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Summary



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