BRUFACE
MA1 Mechanical Engineering

Vibrations and Acoustics
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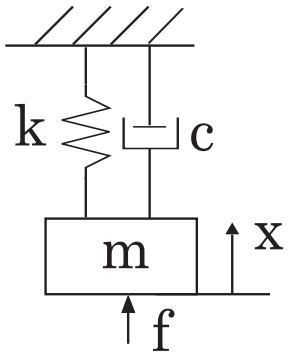
Session 1 : One DOF systems

Exercise 1

Consider the following one-degree-of-freedom (1 DOF) system



- Write the equation of motion in the time domain.

$$m\ddot{x} + c\dot{x} + kx = f$$

  Give the expression of ω_n and of ξ

$$\omega_n = \sqrt{k/m} \qquad \xi = \frac{c}{2\sqrt{mk}}$$

- For this system

  a) Give the expression of the impulse response and represent it using the following numerical values: $m = 1\,kg$, $k = 16\,N/m$, $c = 0.1\,Ns/m$)

$$h(t) = \frac{1}{m\omega_d} e^{-\xi\omega_n t} \sin \omega_d t$$

  with $\omega_d = \omega_n\sqrt{1-\xi^2}$

```
t=linspace(0,100,1000);
k=16; m=1; c=0.1; wn=sqrt(k/m); xi=c/(2*sqrt(k*m));
wd=wn*sqrt(1-xi^2);
h=1/(m*wd)*exp(-xi*wn*t).*sin(wd*t);
figure; set(gca,'Fontsize',15); plot(t,h);
xlabel('time'); ylabel('Impulse response h(t)')
```

b) Give the expression of the harmonic forced response and represent it using the Bode diagram

$$H(\omega) = \frac{X}{F} = \frac{1}{m\left(\omega_n^2 - \omega^2 + 2j\xi\omega\omega_n\right)}$$

```
w=linspace(0,20,10000);
H=1./(m*(wn^2-w.^2+2*j*xi*w*wn));
figure; set(gca,'Fontsize',15);
subplot(2,1,1); plot(w,20*log10(abs(H)));
xlabel('\omega');ylabel('Mag(dB)')
subplot(2,1,2); plot(w,angle(H)*180/pi);
xlabel('\omega');ylabel('Phase(deg)')
```

c) Repeats points a),b) and c) with the following successive values of damping: $c = 0.1\,Ns/m$, $c = 0.5\,Ns/m$, $c = 10\,Ns/m$. What are the corresponding values of ξ? Plot the respective responses on the same plot

```
t=linspace(0,100,1000);
k=16; m=1; c=0.1; wn=sqrt(k/m); ci=[0.1 0.5 10];
color='rgb';

figure; set(gca,'Fontsize',15)
for i=1:length(ci)
c=ci(i);
xi=c/(2*sqrt(k*m)); wd=wn*sqrt(1-xi^2);
h=1/(m*wd)*exp(-xi*wn*t).*sin(wd*t);
plot(t,h,color(i)); hold on;
end

figure; set(gca,'Fontsize',15)
for i=1:length(ci)
c=ci(i);
xi=c/(2*sqrt(k*m)); wd=wn*sqrt(1-xi^2);
H=1./(m*(wn^2-w.^2+2*j*xi*w*wn));
subplot(2,1,1); plot(w,20*log10(abs(H)),color(i)); hold on
xlabel('\omega');ylabel('Mag(dB)')
subplot(2,1,2); plot(w,angle(H)*180/pi,color(i)); hold on
xlabel('\omega');ylabel('Phase(deg)')
end

figure; set(gca,'Fontsize',15)
for i=1:length(ci)
c=ci(i);
xi=c/(2*sqrt(k*m)); wd=wn*sqrt(1-xi^2);
H=1./(m*(wn^2-w.^2+2*j*xi*w*wn));
RE=real(H); IM=imag(H);
plot(RE(:),IM(:),color(i)); hold on
xlabel('Re(H)'); ylabel('Im(H)')
end
```

$$c = 0.01 \rightarrow \xi = 0.0125 \quad c = 0.05 \rightarrow \xi = 0.625 \quad c = 10 \rightarrow \xi = 1.25$$

Exercise 2

Consider the same 1-DOF system as in the previous exercise and a value of $c = 0.1\,Ns/m$. Use Duhamel's integral to compute the response of the system to:

a) A harmonic force of the form $f(t) = sin(\omega t)$ where $\omega = \omega_n$

b) A harmonic force of the form $f(t) = sin(\omega t)$ where $\omega = 0.95\omega_n$

c) A random force generated from a gaussian distribution of mean 0 and variance $\sigma = 1$

```
% Create impulse response
k=1; m=1; xi=0.01;
c=2*xi*sqrt(k*m); % for info only

wn=sqrt(k/m); wd=wn*sqrt(1-xi^2);
t=linspace(0,500*2*pi,20000);
h=exp(-xi*wn*t)/(m*wd).*sin(wd*t);

figure; plot(t,h,'Linewidth',2); xlabel('t'); ylabel('h(t)');
set(gca,'Fontsize',15); set(gca,'XLim',[0 500])
a=get(gcf,'CurrentAxes'); set(a,'Box','off'); set(a,'Linewidth',2)

% Sinusoidal inputs
w=1*wn; f=sin(w*t); u=conv(h,f)*t(2); u=u(1:length(t));

figure; plot(t,f,'Linewidth',2); xlabel('t'); ylabel('f(t)');
set(gca,'Fontsize',15); set(gca,'XLim',[0 500])
a=get(gcf,'CurrentAxes'); set(a,'Box','off'); set(a,'Linewidth',2)

figure; plot(t,u,'Linewidth',2); xlabel('t'); ylabel('x(t)');
set(gca,'Fontsize',15); set(gca,'XLim',[0 500])
a=get(gcf,'CurrentAxes'); set(a,'Box','off'); set(a,'Linewidth',2)

w=.95*wn; f=sin(w*t); u=conv(h,f)*t(2); u=u(1:length(t));

figure; plot(t,f,'Linewidth',2); xlabel('t'); ylabel('f(t)');
set(gca,'Fontsize',15); set(gca,'XLim',[0 500])
a=get(gcf,'CurrentAxes'); set(a,'Box','off'); set(a,'Linewidth',2)

figure; plot(t,u,'Linewidth',2); xlabel('t'); ylabel('x(t)');
set(gca,'Fontsize',15); set(gca,'XLim',[0 500])
a=get(gcf,'CurrentAxes'); set(a,'Box','off'); set(a,'Linewidth',2)

% Random input
f=randn(length(t),1);
u=conv(h,f)*t(2); u=u(1:length(t));

figure; plot(t,f); xlabel('t'); ylabel('f(t)');
set(gca,'Fontsize',15); set(gca,'XLim',[0 500])
a=get(gcf,'CurrentAxes'); set(a,'Box','off'); set(a,'Linewidth',2)
```

```
figure; plot(t,u); xlabel('t'); ylabel('x(t)');
set(gca,'Fontsize',15); set(gca,'XLim',[0 500])
a=get(gcf,'CurrentAxes'); set(a,'Box','off'); set(a,'Linewidth',2)
```