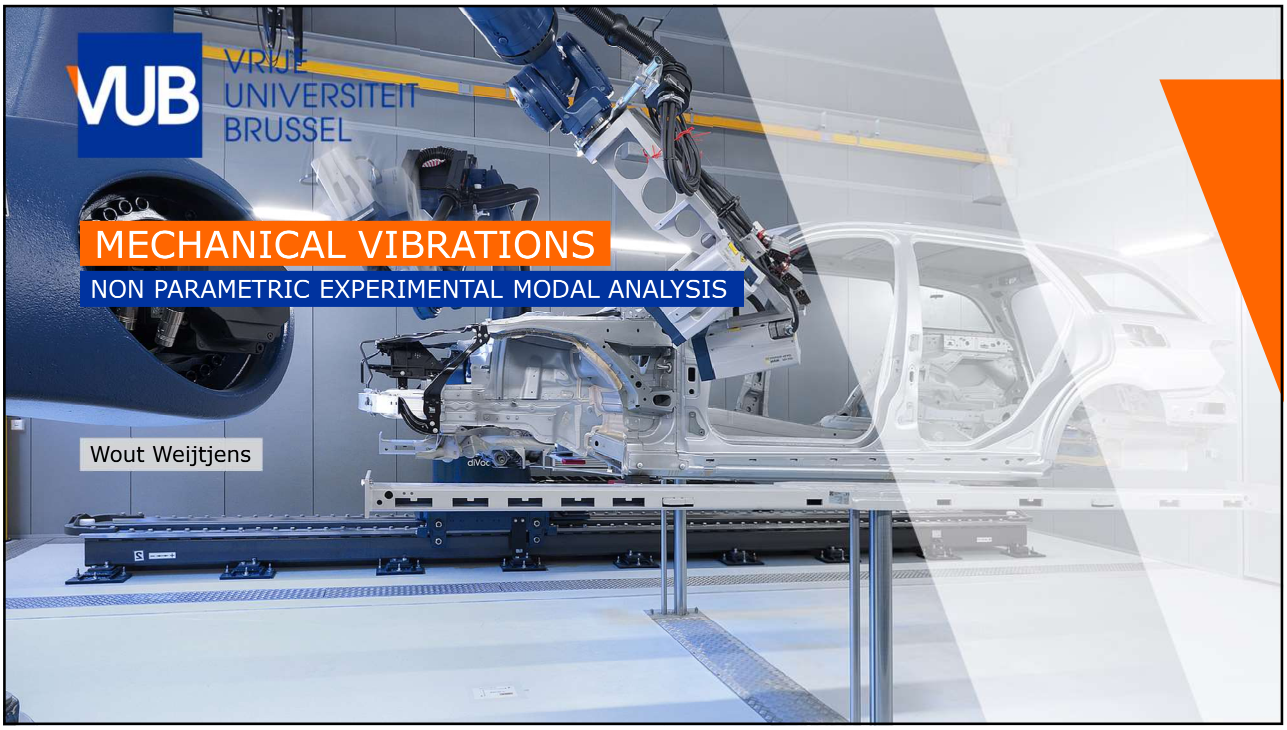

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STARTING

SO WE DID OUR MEASUREMENTS
…
NOW WHAT?

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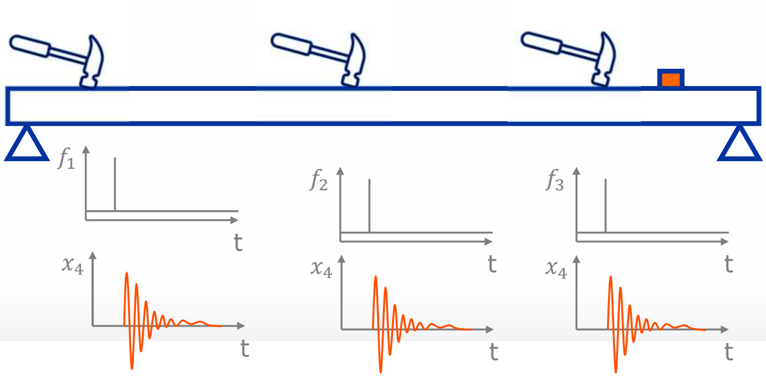
SIGNAL PROCESSING

FROM MEASUREMENTS TO A TRANSFER FUNCTION ESTIMATE

So at this point we measured inputs $f_i(t)$ at N_i locations,

Simultaneously we measured the corresponding system responses $x_o(t)$ at N_o different locations.



SIGNAL PROCESSING

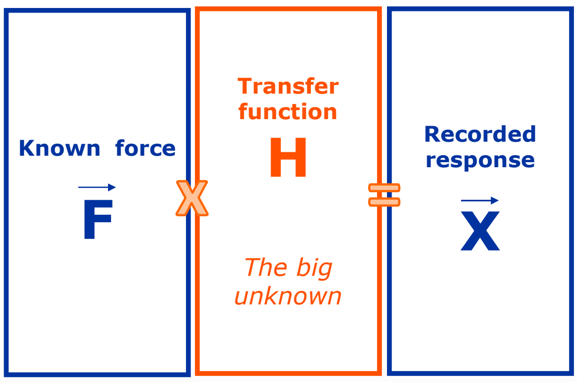
FROM MEASUREMENTS TO A TRANSFER FUNCTION ESTIMATE

So how do we translate these measurements into an estimate of the transfer function?

Easiest is to work in the frequency domain.

Is it then simply?

$$H(\omega) = \frac{X(\omega)}{F(\omega)}$$



SIGNAL PROCESSING

FROM MEASUREMENTS TO A TRANSFER FUNCTION ESTIMATE

So first things first, the transfer equation is not a single parameter to be estimated.

A transfer estimate has to be calculated for **every frequency line**

$$H(\omega) = \frac{X(\omega)}{F(\omega)} \quad \forall\omega$$

SIGNAL PROCESSING

FROM MEASUREMENTS TO A TRANSFER FUNCTION ESTIMATE

So first things first, the transfer equation is not a single parameter to be estimated.

A transfer estimate has to be calculated for **every frequency line**

$$H(\omega) = \frac{X(\omega)}{F(\omega)} \quad \forall\omega$$

But that ignores the presence of noise :

- Mechanical noise
- Non-linearities
- Electrical noise in the instrumentation

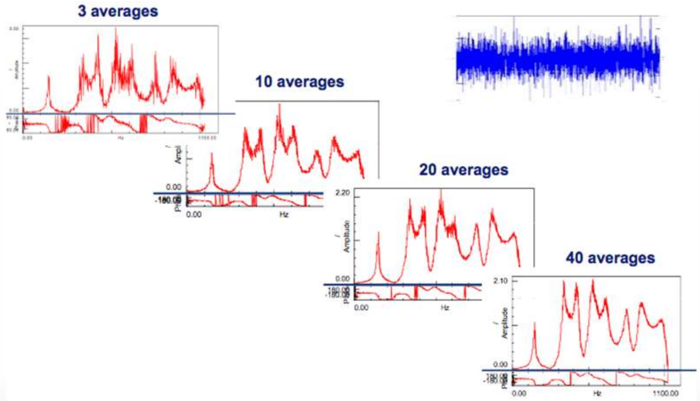
And there is noise on either input, output or both.

$$H(\omega) \approx \frac{X(\omega) + N_X(\omega)}{F(\omega) + N_F(\omega)} \quad \forall\omega$$

SIGNAL PROCESSING

FROM MEASUREMENTS TO A TRANSFER FUNCTION ESTIMATE

So we need to average to get rid of (measurement) noise.

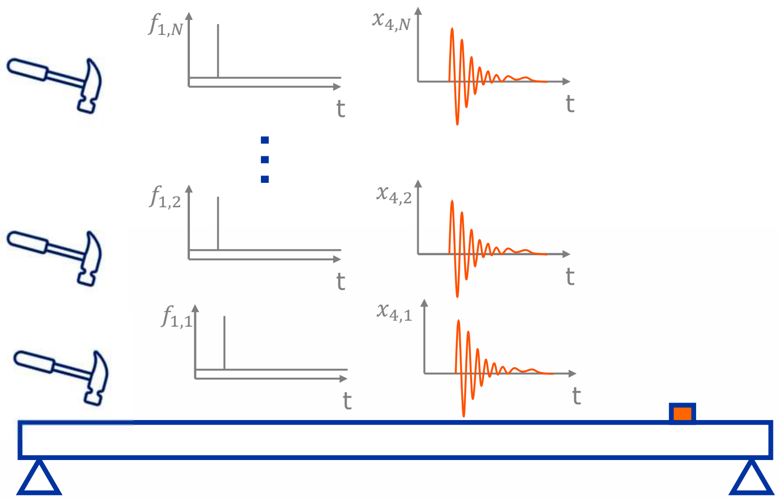


SIGNAL PROCESSING

TO AVERAGE IS TO REPEAT

So in practice to average N_s times, we'll need to repeat each experiment N_s times.

And with each experiment we collect a new instance of the inputs and outputs



N_s : number of samples

NON PARAMETRIC ESTIMATE

AVERAGING PREMISE

Let us start with a SISO system and assume that the input force is random.

$$\hat{F}(\omega) = \frac{1}{N_s}\sum_{k}^{N_s} F_k(\omega) + N_{F,k}(\omega)$$

But $F_{i,k}(\omega)$ is random with random phase and just like a sine summed with its antiphase signal….

$\hat{F}_i(\omega) \rightarrow 0$ when number of averages N_s increases. The same applies to the response , as the response to random is also random.

$$H(\omega) \approx \frac{\sum_k^{N_s} X_k(\omega) + N_{X,k}(\omega)}{\sum_k^{N_s} F_k(\omega_i) + N_{F,k}(\omega)} \quad \forall\omega$$

Will not work

NON PARAMETRIC ESTIMATE

CALCULATION OF THE POWER SPECTRA

Rather to work with the Spectra themselves, we will work with the power spectra

Auto spectra

$$\widehat{G_{XX}}(\omega) = \frac{1}{N_s}\sum_{k}^{N_s} X_k(\omega)X_k^H(\omega) \qquad \widehat{G_{FF}}(\omega_i) = \frac{1}{N_s}\sum_{k}^{N_s} F_k(\omega)F_k^H(\omega)$$

Cross spectra

$$\widehat{G_{XF}}(\omega) = \frac{1}{N_s}\sum_{k}^{N_s} X_k(\omega)F_k^H(\omega)$$

NON PARAMETRIC ESTIMATES

H1 ESTIMATE

The H1 estimate uses the cross-spectrum and the input auto-spectrum

$$\boldsymbol{H}_1(\boldsymbol{\omega}) = \frac{\widehat{\boldsymbol{G}_{XF}}(\omega)}{\widehat{\boldsymbol{G}_{FF}}(\omega)} = \frac{\boldsymbol{G}_{XF}(\omega) + \boldsymbol{G}_{N_XF}(\omega) + \boldsymbol{G}_{XN_F}(\omega) + \boldsymbol{G}_{N_XN_F}(\omega)}{\boldsymbol{G}_{FF}(\omega) + \boldsymbol{G}_{N_FF}(\omega) + \boldsymbol{G}_{FN_F}(\omega) + \boldsymbol{G}_{N_FN_F}(\omega)}$$

If we now assume that **input and output are not correlated to any noise** and **input noise and output noise are uncorrelated,** then for sufficient number of averages.

$$\boldsymbol{H}_1(\boldsymbol{\omega}) = \frac{\widehat{\boldsymbol{G}_{XF}}(\omega)}{\widehat{\boldsymbol{G}_{FF}}(\omega)} = \frac{\boldsymbol{G}_{XF}(\omega)}{\boldsymbol{G}_{FF}(\omega) + \boldsymbol{G}_{N_FN_F}(\omega)}$$

If we now assume that **no input noise**

$$\boldsymbol{H}_1(\boldsymbol{\omega}) = \frac{\widehat{\boldsymbol{G}_{XF}}(\omega)}{\widehat{\boldsymbol{G}_{FF}}(\omega)} = \frac{\boldsymbol{G}_{XF}(\omega)}{\boldsymbol{G}_{FF}(\omega)}$$

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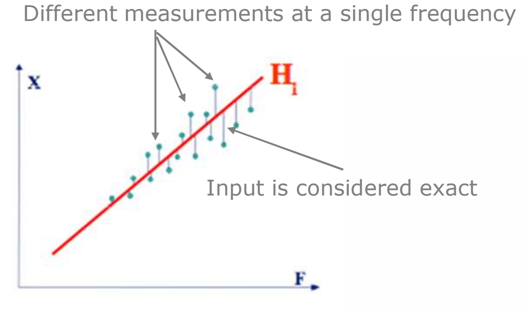
NON PARAMETRIC ESTIMATES

H1 ESTIMATE

The H1 estimate is the equivalent of a **Least squares estimate** of the transfer function



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NON PARAMETRIC ESTIMATES

H1 ESTIMATE

But what if there is noise on the inputs

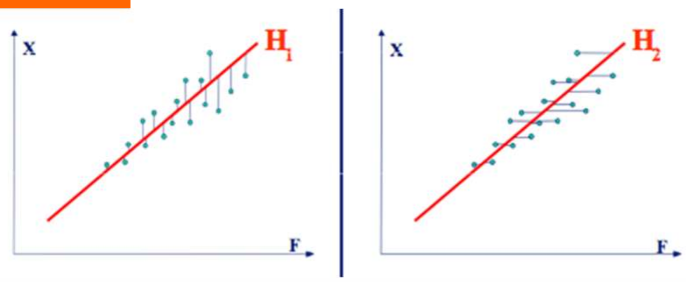
$$H_1(\omega) = \frac{\widehat{G_{XF}}(\omega)}{\widehat{G_{FF}}(\omega)} = \frac{G_{XF}(\omega)}{G_{FF}(\omega) + G_{N_F N_F}(\omega)}$$

Biased outcome, **underestimation** of the transfer function.

This typically manifests itself at frequencies where the amplitude of the force spectrum is low. Often at the resonance frequencies.

NON PARAMETRIC ESTIMATES

H1 AND H2 ESTIMATE

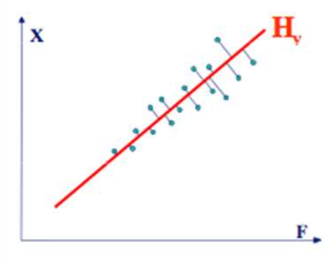


	H1	H2
Formula	$H_1(\omega) = \frac{\widehat{G_{XF}}(\omega)}{\widehat{G_{FF}}(\omega)}$	$H_2(\omega) = \frac{\widehat{G_{XX}}(\omega)}{\widehat{G_{FX}}(\omega)}$
Assumption	No noise on input (F)	No noise on output (X)
Potential issue	Underestimation in presence of input noise	Overestimation in presence of output noise
Potential gain	Better at anti-resonances	Better at resonances

NON PARAMETRIC ESTIMATES

HV ESTIMATE

In the presence of both input and output noise, the Hv estimator is used. Unlike the H1 and H2, the Hv is a **Total Least Squares** estimator.



"When in doubt use Hv"

NON PARAMETRIC ESTIMATES

WHERE ARE WE NOW?

How does this relate with e.g. our hammer test, with an accelerometer at location j

- For every location i out of the N_i locations we will test
  - For every sample k out of the N_s we will collect per location
    - we do the impact,
    - collect the applied force $f_{i,k}(t)$ and the corresponding response $x_{j,k}(t)$
    - Apply the proper windows and transform into to collect the frequency domain $F_{i,k}(\omega)$ and $X_{j,k}(\omega)$
  - We compute the auto and cross power spectra $\widehat{G_{FF}}(\omega)$, $\widehat{G_{XX}}(\omega)$, $\widehat{G_{XF}}(\omega)$ and $\widehat{G_{FX}}(\omega)$

NON PARAMETRIC ESTIMATES

EXTENSION TO MIMO

Up to now we considered the system to be SISO making the powerspectra 0D. However, they are 2D when considering multiple input (Ni) and multiple outputs (No).

$$\widehat{G_{XX}}(\omega) = \frac{1}{N_S}\sum^{N_s} \boldsymbol{X}_j(\omega)\boldsymbol{X}_j^H(\omega)$$

These are now (No x 1) vectors

Dimensions of the powerspectra

G_{XX} : (No x No), G_{FF} : (Ni x Ni), G_{XF} : (No x Ni), G_{FX} : (Ni x No)

$$\overline{H_2}(\omega) = \widehat{G_{XX}}(\omega) \cdot \widehat{G_{FX}}^{\dagger}(\omega)$$

With $\cdot^{\dagger}$ the pseudo inverse

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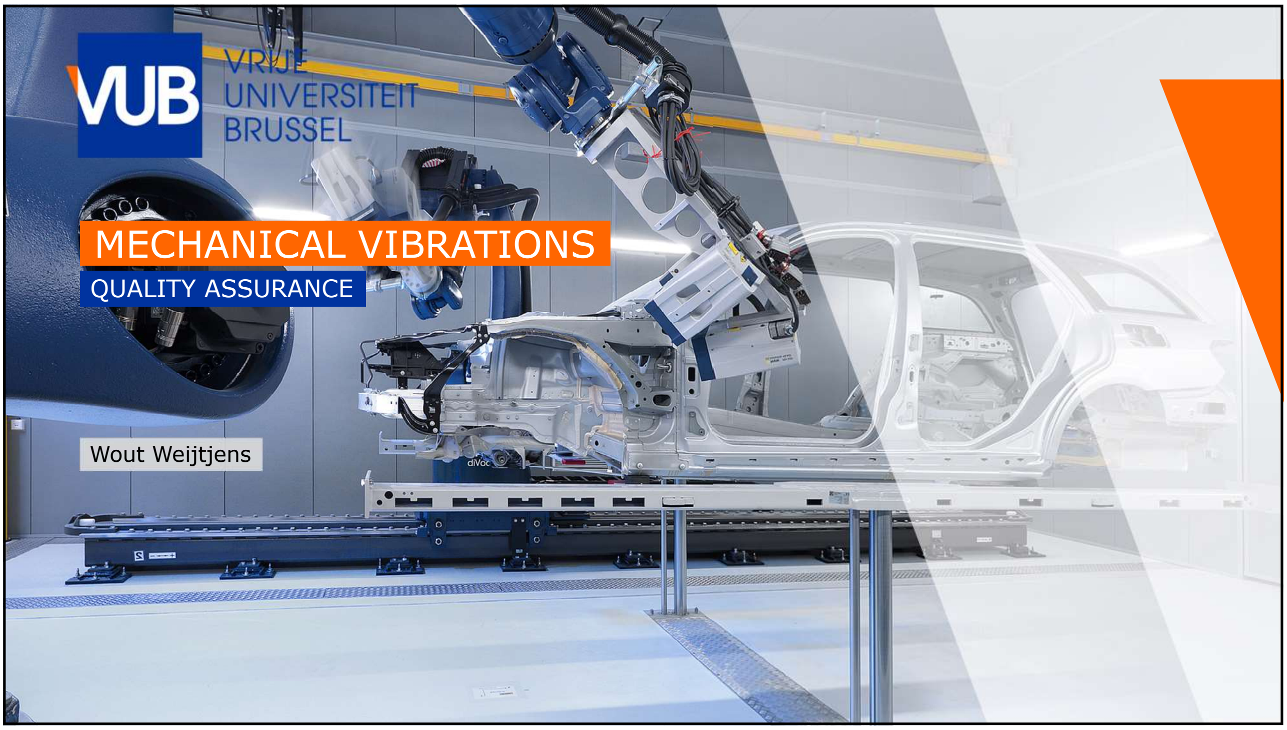

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QUALITY ASSURANCE

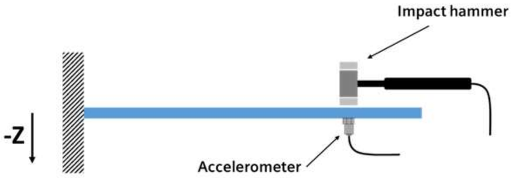
DRIVING POINT

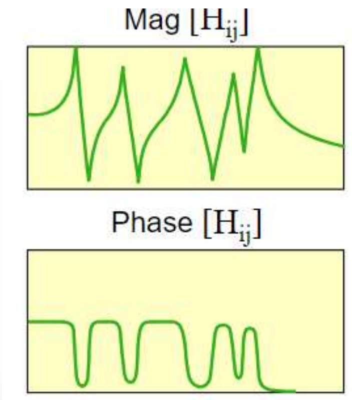
An important (must-do) measurement is the so-called **Driving point (DP) Transfer function,** where input and output location are the same.

- For modal hammer testing : hitting **near** the accelerometer
- For shaker testing : having an output measurement near the input location

DP should have :

- All modes present -> else you installed the accelerometer or shaker in a nodal point!
- Alternating resonances and anti-resonances
- Phase jumping between 0 – 180 degrees





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QUALITY ASSURANCE

COHERENCE

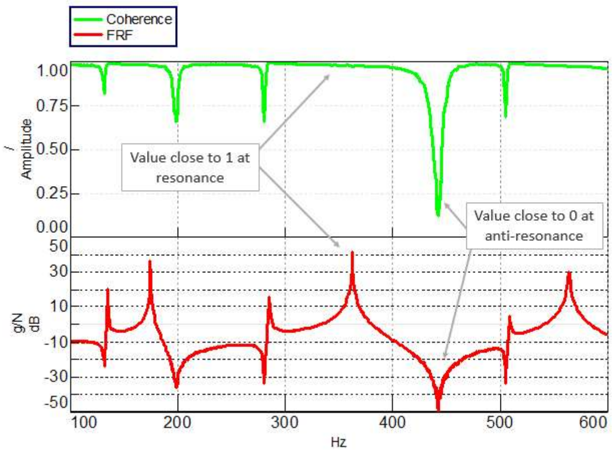
The **coherence** is a great metric to assess the quality of the measurements.

$$\gamma^2(\omega) = \frac{|G_{FX}(\omega)|^2}{G_{XX}(\omega)G_{FF}(\omega)}$$

"How well does input and outputs line up (through linear glasses)"

Coherence should be close to 1, else

- Noise in the measurements
  - Variation in excitation direction (e.g. with hammer)
  - Sensor, cabling issues (e.g. overloads, bad connection)
- Non-linearities
- Leakage



Note that if only one measurement is performed, the coherence will be a value of 1! The value will be one across the entire frequency range – giving the appearance of a "perfect" measurement. This is because at least two FRF measurements need to be take and compared to start to calculate a meaningful coherence function. Don't be fooled!

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QUALITY ASSURANCE
COHERENCE

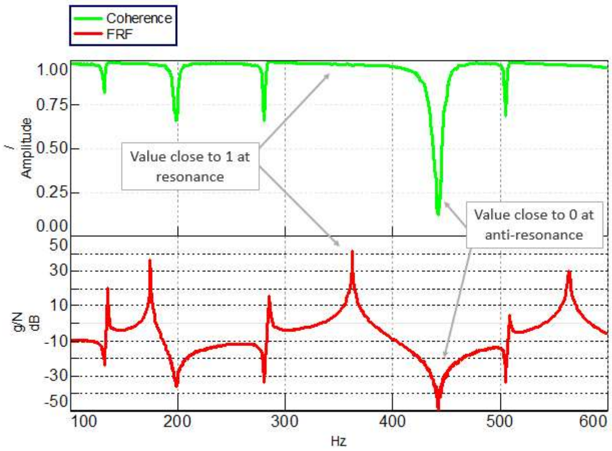
Zero at the anti-resonance?!

This is normal behaviour, the system response is minimal (as expected).

Signal to Noise is poor at these frequencies

-> Input does not match with 'noisy' output

-> Low coherence



QUALITY ASSURANCE
COHERENCE (EXERCISE)

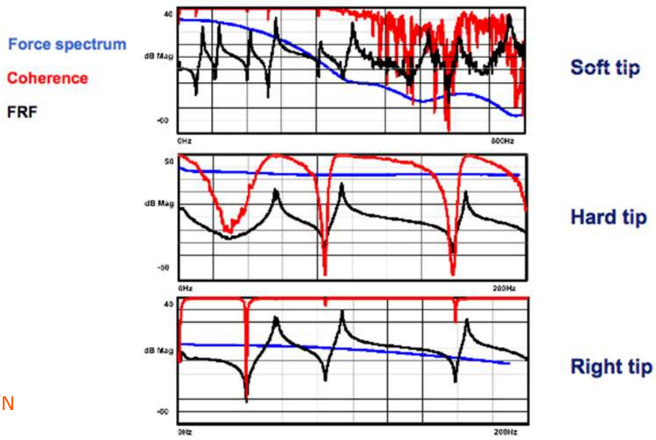
What is the cause of the poor coherence during these impact testing results of :

- High frequency content with the soft tip

  Insufficient energy at high frequencies, poor S/N of the input.

- Area around anti-resonance with hard tip

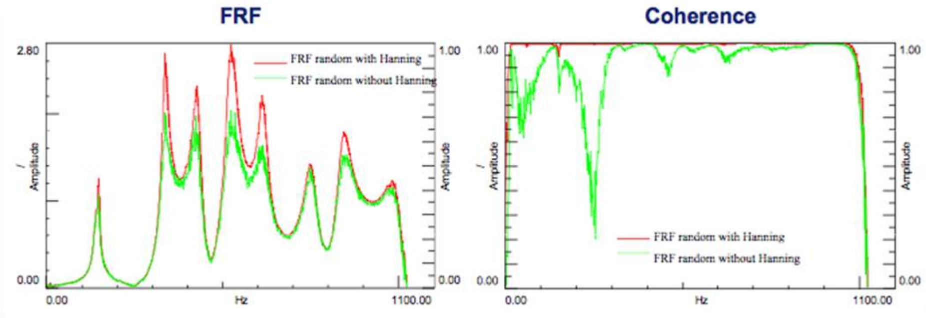
  Insufficient energy injected into the system (short impact). Poor excitation of anti-resonances, poor S/N ratio of output near anti-resonance.



QUALITY ASSURANCE

COHERENCE (EXERCISE)

Impact of leakage is also visible in the coherence



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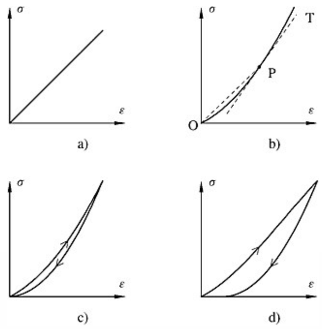
QUALITY ASSURANCE

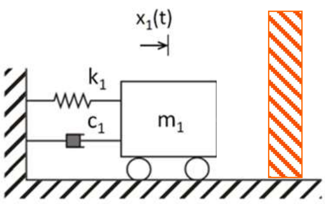
CHECKING LINEARITY



All the theory presented in this course assumes a linear time-invariant system. However, non-linearities are not to be excluded.

Exemplary causes of non-linear behaviour:

- Non-linear material properties e.g. rubbers, plastic
- Geometric constrictions : e.g. hitting a *stopper*
- Geometric deformation (large loads)



Coherence function will indicate presence of non-linearities

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QUALITY ASSURANCE

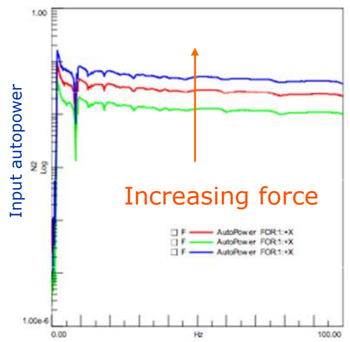
CHECKING LINEARITY

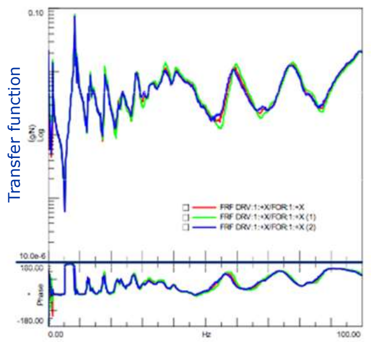
For a linear structure the transfer function is independent of the applied force.

I.e.

$$X(\omega) = H(\omega)F(\omega)$$

$$\alpha X(\omega) = H(\omega)\alpha F(\omega)$$





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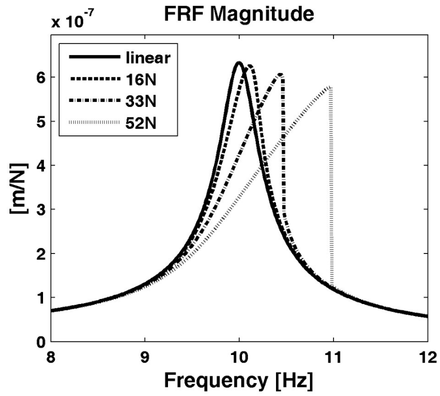
QUALITY ASSURANCE

CHECKING LINEARITY

Textbook example of a non-linear system response under varying loads



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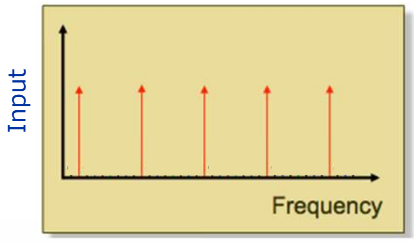
QUALITY ASSURANCE
CHECKING LINEARITY

Advanced strategies are to investigate the 'off-spectral' content when using Multi-sine excitation

- Only excite at red frequency lines
  - So not all frequencies that you can excite.
  - A linear system would only respond on these excited frequencies



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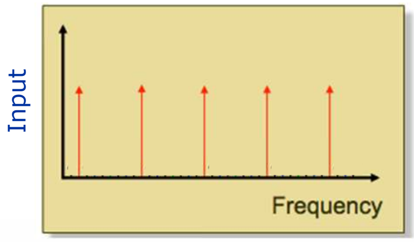
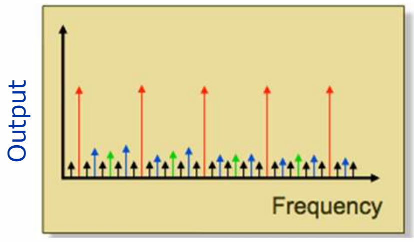
QUALITY ASSURANCE
CHECKING LINEARITY

Advanced strategies are to investigate the 'off-spectral' content when using Multi-sine excitation

- Only excite at red frequency lines
  - So not all frequencies that you can excite.
  - A linear system would only respond on these excited frequencies
- Non-linear response manifests at the non-excited frequencies





Read more : Schoukens, J., Rolain, Y., Swevers, J., & De Cuyper, J. (2000). Simple methods and insights to deal with non-linear distortions in FRF-measurements. *Mechanical Systems and Signal Processing*, *14*(4), 657-666.

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